Geometry

This course introduces students to basic theorems of Euclidean plane geometry and their applications, and explores both plane and solid geometric figures. Students learn how to prove theorems by the axiomatic method and to use these theorems in solving a variety of problems. Students also learn how to accomplish a variety of geometric constructions. The following book is required for this course:

- *Geometry* (includes answers to Chapter Reviews), by Larson, Boswell, and Stiff, McDougall Littell/Houghton Mifflin publishers, 2001

Contents of *Geometry* textbook:

7.4 Translations and Vectors
7.5 Glide Reflections and Compositions
7.6 Frieze Patterns

8. Similarity

8.1 Ratio and Proportion
8.2 Problem Solving in Geometry with Proportions
8.3 Similar Polygons
8.4 Similar Triangles
8.5 Proving Triangles Are Similar
8.6 Proportions and Similar Triangles
8.7 Dilations

9. Right Triangles and Trigonometry

9.1 Similar Right Triangles
9.2 The Pythagorean Theorem
9.3 The Converse of the Pythagorean Theorem
9.4 Special Right Triangles
9.5 Trigonometric Ratios
9.6 Solving Right Triangles
9.7 Vectors

10. Circles

10.1 Tangents to Circles
10.2 Arcs and Chords
10.3 Inscribed Angles
10.4 Other Angle Relationships in Circles
10.5 Segment Lengths in Circles
10.6 Equations of Circles
10.7 Locus

11. Area of Polygons and Circles

11.1 Angle Measures in Polygons
11.2 Areas of Regular Polygons
11.3 Perimeters and Areas of Similar Figures
11.4 Circumference and Arc Length
11.5 Areas of Circles and Sectors

Chapter 2.1

Goal 1: Recognizing Conditional Statements

In this lesson you will study a type of logical statement called a conditional statement. A **conditional statement** has two parts, a *hypothesis* and a *conclusion*. When the statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**. Here is an example:

If **it is noon in Georgia**, then *it is 9 A.M. in California*.
hypothesis *conclusion*

Example 1: Rewriting in If-Then Form

Rewrite the conditional statement in *if-then form*.

a. Two points are collinear if they lie on the same line.

b. If a fish is a shark, then it has a boneless skeleton.

c. If a number is divisible by 9, then it is divisible by 3.

Conditional statements can be either true or false. To show that a conditional statement is true, you must present an argument that the conclusion follows for *all* cases that fulfill the hypothesis. To show that a conditional statement is false, describe a single counterexample that shows the statement is not always true.

Example 2: Writing a Counterexample

Write a counterexample to show that the following conditional statement is false.

If $x^2 = 16$, then $x = 4$.

Solution

As a counterexample, let $x = -4$. The hypothesis is true, because $(-4)^2 = 16$. However, the conclusion is false. This implies that the given conditional statement is false.

The **converse** of a conditional statement is formed by switching the hypothesis and conclusion. Here is an example.

Statement: If you see lightning, then you hear thunder.

Converse: If you hear thunder, then you see lightning.

Example 3: Writing the Converse of a Conditional Statement

Write the converse of the following conditional statement.

Statement: If two segments are congruent, then they have the same length.

Solution

Converse: If two segments have the same length, then they are congruent.

A statement can be altered by **negation**, that is, by writing the negative of the statement. Here are some examples.

STATEMENT	NEGATION
$m\angle A = 30°$	$m\angle A \neq 30°$
$\angle A$ is acute.	$\angle A$ is not acute.

When you negate the hypothesis and conclusion of a conditional statement, you form the **inverse**. When you negate the hypothesis and conclusion of the converse of a conditional statement, you form the **contrapositive**.

Original	If $m\angle A = 30°$, then $\angle A$ is acute.	true
Inverse	If $m\angle A \neq 30°$, then $\angle A$ is not acute.	false
Converse	If $\angle A$ is acute, then $m\angle A = 30°$.	false
Contrapositive	If $\angle A$ is not acute, then $m\angle A \neq 30°$.	true

When two statements are both true or both false, they are called **equivalent statements**. A conditional statement is equivalent to its contrapositive. Similarly, the inverse and converse of any conditional statement are equivalent. This is shown in the table above.

Example 4: Writing an Inverse, Converse, and Contrapositive

Write the **(a)** inverse, **(b)** converse, and **(c)** contrapositive of the statement.

If there is snow on the ground, then flowers are not in bloom.

Solution

a. **Inverse:** If there is no snow on the ground, then flowers are in bloom.

b. **Converse:** If flowers are not in bloom, then there is snow on the ground.

c. **Contrapositive:** If flowers are in bloom, then there is no snow on the ground.

Goal 2: Using Point, Line, and Plane Postulates

In Chapter 1, you studied four postulates.

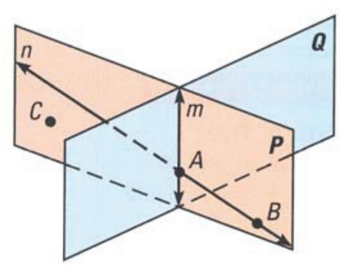
Ruler Postulate	(Lesson 1.3, page 17)
Segment Addition Postulate	(Lesson 1.3, page 18)
Protractor Postulate	(Lesson 1.4, page 27)
Angle Addition Postulate	(Lesson 1.4, page 27)

Remember that postulates are assumed to be true – they form the foundation on which other statements (called *theorems*) are built.

Point, Line, and Plane Postulates	
POSTULATE 5	Through any two points there exists exactly one line.
POSTULATE 6	A line contains at least two points.
POSTULATE 7	If two lines intersect, then their intersection is exactly one point.
POSTULATE 8	Through any three noncollinear points there exists exactly one plane.
POSTULATE 9	A plane contains at least three noncollinear points.
POSTULATE 10	If two points lie in a plane, then the line containing them lies in the plane.
POSTULATE 11	If two planes intersect, then their intersection is a line.

Example 5: Identifying Postulates

Use the diagram at right to give examples of Postulates 5 through 11.



Solution

a. Postulate 5: There is exactly one line (line *n*) that passes through the points *A* and *B*.

b. Postulate 6: Line *n* contains at least two points. For instance, line *n* contains the points *A* and *B*.

c. Postulate 7: Lines *m* and *n* intersect at point *A*.

d. Postulate 8: Plane *P* passes through the noncollinear points *A*, *B*, and *C*.

e. Postulate 9: Plane *P* contains at least three noncollinear points, *A*, *B*, and *C*.

f. Postulate 10: Points *A* and *B* lie in plane *P*. So, line *n*, which contains points *A* and *B*, also lies in plane *P*.

g. Postulate 11: Planes *P* and *Q* intersect. So, they intersect in a line, labeled in the diagram as line *m*.

Example 6: Rewriting a Postulate

a. Rewrite Postulate 5 in if-then form.

b. Write the inverse, converse, and contrapositive of Postulate 5.

Solution

a. Postulate 5 can be written in if-then form as follows:

If two points are distinct, then there is exactly one line that passes through them.

b. **Inverse:** If two points are not distinct, then it is not true that there is exactly one line that passes through them.

Converse: If exactly one line passes through two points, then the two points are distinct.

Contrapositive: If it is not true that exactly one line passes through two points, then the two points are not distinct.

Example 7: Using Postulates and Counterexamples

Decide whether the statement is *true* or *false*. If it is false, give a counterexample.

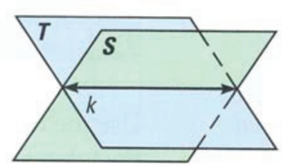
a. A line can be in more than one plane.

b. Four noncollinear points are always coplanar.

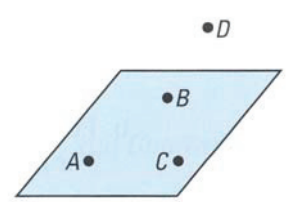
c. Two nonintersecting lines can be noncoplanar.

Solution

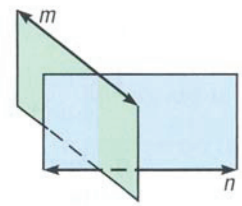
a. In the diagram at the right, line k is in plane S and line k is in plane T.
So, it is *true* that a line can be in more than one plane.



b. Consider the points A, B, C, and D at the right. The points A, B, and C lie in a plane, but there is no plane that contains all four points.
So, as shown in the counterexample at the right, it is *false* that four noncollinear points are always coplanar.



c. In the diagram at the right, line *m* and line *n* are nonintersecting and are also noncoplanar.



d. So, it is *true* that two nonintersecting lines can be noncoplanar.