

Algebra I

This course covers topics typically treated in a first-year algebra course. It includes evaluation of expressions involving signed numbers, exponents and roots, properties of the real numbers, absolute value and equations and inequalities involving absolute value, scientific notation, unit conversions, solution of equations in one unknown and solution of simultaneous equations, the algebra of polynomials and rational expressions, word problems requiring algebra for their solution, graphical solution of simultaneous equations, Pythagorean theorem, algebraic proofs, functions and functional notation, solution of quadratic equations via factoring and completing the square, direct and inverse variation, and exponential growth. The following book is required for this course:

- *Saxon Algebra I, An Incremental Development* (textbook and Home Study Packet) Third Edition, Saxon Publishers, by John H. Saxon, Jr., 1997

Contents of the *Saxon Algebra I* textbook and Home Study Packet:

Preface

1. Addition and Subtraction of Fractions; Lines and Segments
2. Angles; Polygons; Triangles; Quadrilaterals
3. Perimeter; Circumference
4. Review of Arithmetic
5. Sets; Absolute Value; Addition of Signed Numbers
6. Rules for Addition; Adding More Than Two Numbers; Inserting Parentheses Mentally; Definition of Subtraction
7. The Opposite of a Number; Simplifying More Difficult Notations
8. Area
9. Rules for Multiplication of Signed Numbers; Inverse Operations; Rules for Division of Signed Numbers; Summary
10. Division by Zero; Exchange of Factors in Multiplication; Conversions of Area
11. Reciprocal and Multiplicative Inverse; Order of Operations; Identifying Multiplication and Addition
12. Symbols of Inclusion; Order of Operations
13. Multiple Symbols of Inclusion; More on Order of Operations; Products of Signed Numbers

14. Evaluation of Algebraic Expressions
15. Surface Area
16. More Complicated Evaluations
17. Factors and Coefficients; Terms; The Distributive Property
18. Like Terms; Addition of Like Terms
19. Exponents; Powers of Negative Numbers; Roots; Evaluation of Powers
20. Volume
21. Product Rule for Exponents; Addition of Like Terms with Exponents
22. Review of Numerical and Algebraic Expressions; Statements and Sentences; Conditional Equations
23. Equivalent Equations; Additive Property of Equality
24. Multiplicative Property of Equality
25. Solution of Equations
26. More Complicated Equations
27. More on the Distributive Property; Simplifying Decimal Equations
28. Fractional Parts of Numbers; Functional Notation
29. Negative Exponents; Zero Exponents
30. Algebraic Phrases; Decimal Parts of a Number
31. Equations with Parentheses
32. Word Problems
33. Products of Prime Factors; Statements about Unequal Quantities
34. Greatest Common Factor
35. Factoring the Greatest Common Factor; Canceling
36. Distributive Property of Rational Expressions that Contain Positive Exponents; Minus Signs and Negative Exponents
37. Inequalities; Greater Than and Less Than; Graphical Solutions of Inequalities
38. Ratio Problems
39. Trichotomy Axiom; Negated Inequalities; Advanced Ratio Problems
40. Quotient Rule for Exponents; Distributive Property of Rational Expressions that Contain Negative Exponents
41. Addition of Like Terms in Rational Expressions; Two-Step Problem
42. Solving Multivariable Equations
43. Least Common Multiple; Least Common Multiples of Algebraic Expressions
44. Addition of Rational Expressions with Equal Denominators; Addition of Rational Expressions with Unequal Denominators
45. Range, Median, Mode, and Mean
46. Conjunctions
47. Percents Less Than 100; Percents Greater Than 100
48. Polynomials; Degree; Addition of Polynomials
49. Multiplication of Polynomials
50. Polynomial Equations; Ordered Pairs; Cartesian Coordinate System
51. Graphs of Linear Equations; Graphs of Vertical and Horizontal Lines;
52. More on Addition of Rational Expressions with Unequal Denominators; Overall Average
53. Power Rule for Exponents
54. Substitution Axiom; Simultaneous Equations; Solving Simultaneous Equations by

- Substitution
55. Complex Fractions; Division Rule for Complex Fractions
 56. Finite and Infinite Sets; Membership in a Set; Rearranging Before Graphing
 57. Addition of Algebraic Expressions with Negative Exponents
 58. Percent Word Problems
 59. Rearranging Before Substitution
 60. Geometric Solids; Prisms and Cylinders
 61. Subsets; Subsets of the Set of Real Numbers
 62. Square Roots; Higher Order Roots; Evaluating Using Plus or Minus
 63. Product of Square Roots Rule; Repeating Decimals
 64. Domain; Additive Property of Inequality
 65. Addition of Radical Expressions; Weighted Average
 66. Simplification of Radical Expressions; Square Roots of Large Numbers
 67. Review of Equivalent Equations; Elimination
 68. More About Complex Fractions
 69. Factoring Trinomials
 70. Probability
 71. Trinomials with Common Factors; Subscripted Variables
 72. Factors That Are Sums; Pyramids and Cones
 73. Factoring the Difference of Two Squares; Probability Without Replacement
 74. Scientific Notation
 75. Writing the Equation of a Line
 76. Consecutive Integers
 77. Consecutive Odd and Consecutive Even Integers; Fraction and Decimal Word Problems
 78. Rational Equations
 79. Systems of Equations with Subscripted Variables
 80. Operations with Scientific Notation
 81. Graphical Solutions; Inconsistent Equations; Dependent Equations
 82. Evaluating Functions; Domain and Range
 83. Coin Problems
 84. Multiplication of Radicals; Functions
 85. Stem-and-Leaf Plots; Histograms
 86. Division of Polynomials
 87. More on Systems of Equations; Tests for Functions
 88. Quadratic Equations; Solution of Quadratic Equations by Factoring
 89. Value Problems
 90. Word Problems with Two Statements of Equality
 91. Multiplicative Property of Inequality; Spheres
 92. Uniform Motion Problems About Equal Distances
 93. Products of Rational Expressions; Quotients of Rational Expressions
 94. Uniform Motion Problems of the Form $D_1 + D_2 = N$
 95. Graphs of Non-Linear Functions; Recognizing Shapes of Various Non-Linear Functions
 96. Difference of Two Squares Theorem
 97. Angles and Triangles; Pythagorean Theorem; Pythagorean Triples
 98. Distance Between Two Points; Slope Formula

99. Uniform Motion-Unequal Distances
100. Place Value; Rounding Numbers
101. Factorable Denominators
102. Absolute Value Inequalities
103. More on Rational Equations
104. Abstract Rational Equations
105. Factoring by Grouping
106. Linear Equations; Equation of a Line Through Two Points
107. Line Parallel to a Given Line; Equation of a Line with a Given Slope
108. Square Roots Revisited; Radical Equations
109. Advanced Trinomial Factoring
110. Vertical Shifts; Horizontal Shifts; Reflection About the x Axis; Combinations of Shifts and Reflections
111. More On Conjunctions; Disjunctions
112. More on Multiplications of Radical Expressions
113. Direct Variation; Inverse Variation
114. Exponential Key; Exponential Growth; Using the Graphing Calculator to Graph Exponential Functions
115. Linear Inequalities
116. Quotient Rule for Square Roots
117. Direct and Inverse Variation Squared
118. Completing the Square
119. The Quadratic Formula; Use of the Quadratic Formula
120. Box-and-Whisker Plots

Lesson 32

Word Problems

To solve word problems, we look for statements in the problems that describe equal quantities. Then we use algebraic phrases and equals signs to write equations that make the same statements of equality. We will begin by solving problems that contain only one statement of equality. These problems require that we write only one equation. Later, we will encounter problems that contain more than one statement of equality. These problems will require more than one equation for their solution.

We will avoid the use of the letters x and y in writing these equations. We will try to use variables whose meaning is easy to remember. The problems in this lesson discuss some unknown number. We will use the letter N to represent the unknown number.

example 32.1 The sum of twice a number and 13 is 75. Find the number.

solution We will use N to represent the unknown number. **The word *is* means “equal to.” Thus, the sum of twice a number and 13 equals 75.**

$$2N + 13 = 75 \text{ equation}$$

We can solve this equation by adding -13 to both sides and then dividing both sides by 2.

$$\begin{array}{r} 2N + 13 = 75 \text{ equation} \\ - 13 \quad - 13 \text{ added } -13 \text{ to both sides} \\ \hline 2N = 62 \\ N = \mathbf{31} \text{ divided both sides by } 2 \end{array}$$

Solutions to word problems should always be checked to see if they really do solve the problem.

$$2(31) + 13 = 75 \rightarrow 62 + 13 = 75 \rightarrow 75 = 75 \text{ Check}$$

example 32.2 Find a number such that 13 less than twice the number is 137.

solution We will use N to represent the unknown number. Then twice the unknown number is $2N$ and 13 less than that is $2N - 13$.

$$\begin{array}{r} 2N - 13 = 137 \text{ equation} \\ + 13 \quad + 13 \text{ added } 13 \text{ to both sides} \\ \hline 2N = 150 \\ N = \mathbf{75} \text{ divided both sides by } 2 \end{array}$$

$$2(75) - 13 = 137 \rightarrow 150 - 13 = 137 \rightarrow 137 = 137 \quad \text{Check}$$

example 32.3 Find a number such that if 5 times the number is decreased by 14, the result is twice the opposite of the number.

solution If we use N for the number, then $2(-N)$ will represent twice the opposite of the number.

$$\begin{array}{ll} 5N - 14 = 2(-N) & \text{equation} \\ 5N - 14 = -2N & \text{multiplied} \\ \frac{2N + 14}{7N = 14} & \text{added } 2N + 14 \text{ to both sides} \\ N = 2 & \text{divided both sides by } 7 \end{array}$$

$$5(2) - 14 = 2(-2) \rightarrow 10 - 14 = -4 \rightarrow -4 = -4 \quad \text{Check}$$

example 32.4 Find a number which decreased by 18 equals 5 times its opposite.

solution Again we use N for the number and $-N$ for its opposite.

$$\begin{array}{ll} N - 18 = 5(-N) & \text{equation} \\ N - 18 = -5N & \text{multiplied} \\ \frac{5N + 18}{6N = 18} & \text{added } 5N + 18 \text{ to both sides} \\ N = 3 & \text{divided both sides by } 6 \end{array}$$

$$3 - 18 = 5(-3) \rightarrow 3 - 18 = -15 \rightarrow -15 = -15 \quad \text{Check}$$

example 32.5 We get the same result if we multiply a number by 3 *or* if we multiply the number by 5 and then add 2. Find the number.

solution The statement of the problem leads to the following equation.

$$\begin{array}{ll} 3N = 5N + 2 & \text{equation} \\ \frac{-5N}{-2N = 2} & \text{added } -5N \text{ to both sides} \\ N = -1 & \text{divided both sides by } -2 \end{array}$$

$$3(-1) = 5(-1) + 2 \rightarrow -3 = -5 + 2 \rightarrow -3 = -3 \quad \text{Check}$$

practice

- Four times a number decreased by 8 equals 92. Find the number. Check your answer.
- If the product of 4 and a number is decreased by 12, the result is twice the opposite of the number. Find the number. Check your answer.