

# Grade 7 Math

## Oak Meadow Coursebook

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# Lesson 1 Mean, Median, Mode, and Range; Exponents; and Order of Operations



## Mental Math

Mental math games are like gymnastics for your brain. They help your brain warm up and develop a flexibility of thinking that will benefit you in all areas of life. There's only one rule for mental math: do all the calculations in your head.

You'll find two variations of each mental math game, but you are highly encouraged to make up your own variations and to involve friends and family members. Feel free to repeat games you like or change ones that don't work well for you. Spend a few minutes doing mental math games before each math session and you will find your brain limber and ready to learn.

**Version 1:** Count to 100 by 2s, and then count back down to zero. Next, count by 3s, ending on the number right before or right after 100 (you can “turn back” at either 99 or 102). Next, count up and down by 4s, then 5s, and so on, all the way up to 10s. Some of the numbers will feel very easy, like counting by 2s, 5s, and 10s, but others will be more challenging, especially going backwards. Like most mental math games, this is a very simple exercise that really makes you think.

**Version 2:** Count to 100 by 2s, then count back down from 100 to zero by 3s—this is harder because you are counting backwards by 3s but not by multiples of 3, because 100 isn't a multiple of 3 (99 is). Whatever number you land on closest to zero, you'll begin counting forward again from that number, this time counting by 4s; then you'll count down by 5s from whatever number you end on that is closest to 100. You'll continue counting up and down, ending and beginning on different numbers each time rather than starting at zero and ending at 100 like the first game. For instance, after counting forward to 100 by 2s, you'll count down from there by 3s: 100, 97, 94, 91, 88, etc., all the way down to 1, and then begin counting forward by 4s from 1: 1, 5, 9, 13, 17, etc. up to 101, then count by 5s down from there: 101, 96, 91, 86, 81, etc.

See if you can keep it going all the way up to counting by 10s—and notice what patterns might emerge along the way.

## Skills Check

Complete the following worksheet to brush up on skills you already know. Take note of any that you are having trouble with, and spend more time working on them so you feel confident moving forward.

- Lesson 1 Skills Check

## New Skills

We'll begin this course by reviewing skills you probably already know. You'll find a lot of information in the first few lessons, but don't let that worry you since it covers things you already know. Hopefully, you'll find this review helps you to refresh your skills and gain more confidence with them.

Because they've been covered before, we'll move pretty quickly, but if you come to a skill that you find confusing or one that is new to you, slow down and take your time learning it. Practice as much as you need to in order to feel confident with that skill before moving to the next lesson.

We'll begin by looking at the following topics:

- Finding Mean, Median, Mode, and Range
- Exponents
- Order of Operations

## Finding Mean, Median, Mode, and Range

We'll begin the year reviewing some concepts you probably remember. Let's first look at four ways to understand information about sets of numbers:

**Mean:** Also called the average, the mean calculates what the amount would be if there were an equal number of items in each group.

**Median:** When the numbers are arranged in order from lowest to highest, the number at the midpoint of the set is called the median.

**Mode:** The number that occurs most frequently in any set of numbers is the mode.

**Range:** The difference between the lowest and the highest number in the set.

Let's look at each one separately, starting with the mean. The mean, or average, tells us what the amount would be if everything were equal. Say you have two sets of colored pencils, one with 12 pencils and one with 16. The mean gives us a way to figure out how to balance out the numbers so each group or set would have the same amount.

**Mean is calculated by adding up all the numbers in the set of numbers and dividing the total by how many numbers there are.**

If we added up the two sets of colored pencils, we'd get the total number of pencils:  $12 + 16 = 28$ . By dividing that by the number of sets (2), we find the mean.  $28 \div 2 = 14$ , so we can say there is a mean of 14 colored pencils per set. As you can see, no set actually *has* 14 pencils; the mean just gives us a way to look at how many each set *would have* if everything were equal.

**Example:** The Johnson family has been collecting coins in three different jars. After one month, one jar holds \$3.35, another jar holds \$4.21, and the third jar holds \$1.83. What is the mean amount of money in each jar?



**To find the mean:**

**Step 1:** Add the amounts in each group to get the total.

$$\begin{array}{r} \$3.35 \\ \$4.21 \\ + \$1.83 \\ \hline \$9.39 \end{array}$$

**Step 2:** Divide the total by the number of groups.

There are 3 jars in all, so we divide the total by 3 to find the mean.

$$\$9.39 \div 3 = \$3.13$$

The mean amount of money in each jar is \$3.13.

Remember, whenever you are answering a word problem, first you calculate the answer mathematically, and then you report the answer in a complete sentence.

Now let's look at how to calculate median and what it shows us.

**Median is calculated by arranging the numbers in order from lowest to highest, and identifying the number at the midpoint of the set. If there are two numbers at the midpoint, the median is found by calculating the mean (or average) of those two numbers.**

Let's look at a couple of examples to see how this works.

**Example:** Five homes were sold in Centerville last month for the following prices:

\$102,000

\$99,000

\$120,000

\$117,000

\$115,000

What is the median price of the homes sold in Centerville last month?

**To find the median:**

**Step 1:** Write the set of numbers in order from lowest to highest:

\$99,000

\$102,000

\$115,000

\$117,000

\$120,000

**Step 2:** The median is the number in the exact middle.

The median price of homes sold last month in Centerville is \$115,000.

Finding the middle number is easy if there is an odd number of items in the set, but where is the midpoint when there is an even number of items?

**Example:** Jessie's baseball team scored the following number of points in games this season: 10, 7, 3, 8, 5, 2. Based on this information (or *data*), what was the median number of points scored?

**Step 1:** Order the numbers in the set from lowest to highest.

2, 3, 5, 7, 8, 10

**Step 2:** Find the number in the middle. In this case, there are two numbers in the middle: 5 and 7. So we average those two numbers (the mean of 5 and 7 is 6) to find the median for the whole set of numbers.

The median number of points scored this season is 6. Did they ever actually score 6 points in a game? No, but the median is 6 because that's the midpoint of all their scores for the season.

You can see that we sometimes have to use mean to find the median, but the two processes are different and give us different ways of looking at data. When you calculate both mean and median for a set of numbers, you'll almost always end up with different numbers because mean and median are showing different things.

Next, let's look at mode. You can remember what mode is because it sounds like the word *most*, which is exactly what mode is: the number that appears most frequently in a set.

**We calculate mode by ordering the numbers from lowest to highest, and identifying which number occurs the most. You might have more than one mode or none at all.**

If a set of numbers has no repeat numbers, there is no mode because no number occurs most. On the other hand, if more than one number occurs the most number of times, you can have more than one mode. Let's look at two examples:

**Example:** Find the mode for the following set of numbers:

34, 33, 33, 36, 31, 36, 39, 33

**To find the mode:**

**Step 1:** Arrange the numbers in order from lowest to highest.

31, 33, 33, 33, 34, 36, 36, 39

**Step 2:** Identify the number that occurs the most.

The number 33 occurs more times than any other number, so the mode is 33.

**Example:** Find the mode for the following set of numbers:

2010, 2008, 2011, 2008, 2010, 2009

**Step 1:** Arrange the numbers in order from lowest to highest.

2008, 2008, 2009, 2010, 2010, 2011

**Step 2:** Identify the number that occurs the most.

In this set of numbers, both 2008 and 2010 appear twice, which is more than any other number. The mode for this set is 2008 and 2010.

Finally, let's calculate the range. This is simply the difference between the highest and lowest value in a set of numbers.

**To calculate range, we determine the highest and lowest values in the set of numbers and then subtract the lowest number from the highest to get the range. If all the numbers are identical, the range is zero.**

**Example:** What is the range of prices for the five houses sold in Centerville in the past month?

Here is the original data set:

\$102,000	\$117,000
\$99,000	\$115,000
\$120,000	

**To find the range:**

**Step 1:** Identify the lowest and highest numbers (it might help to put them in order first).

Lowest sales price: \$99,000

Highest sales price: \$120,000

**Step 2:** Subtract the lowest number from the highest.

$$\$120,000 - \$99,000 = \$21,000$$

The range of prices for homes sold in Centerville last month is \$21,000.

Since we often find data given in several ways at once, let's look at calculating mean, median, mode, and range on one set of numbers.

**Example:** Find the mean, median, mode and the range for the following set of numbers:

$$127, 132, 112, 113, 139, 125, 127$$

**Step 1:** Arrange the numbers in order from lowest to highest.

$$112, 113, 125, 127, 127, 132, 139$$

**Step 2:** Find the mean by adding the numbers together and dividing by how many numbers there are.

The numbers total 875. There are 7 numbers in all, so we divide 875 by 7 to get the mean:  $875 \div 7 = 125$ . The mean is 125.

**Step 3:** Find the median by finding the middle number. The middle number is 127, so that's the median. It doesn't matter that there are two numbers the same—there is only one middle number in this set, and that's 127.

**Step 4:** Find the mode by identifying the number that occurs the most. In this set, 127 occurs twice, so that's the mode.

**Step 5:** Find the range by subtracting the lowest number from the highest.

$$139 - 112 = 27$$

The range is 27.

Here are all the answers for this problem:

Mean: 125

Median: 127

Mode: 127

Range: 27

You can see that sometimes the mean, median, mode, or range will end up being the same number even though they are all calculated differently and represent different things.

## Exponents

Sometimes we need to multiply a number by itself, and an *exponent* can tell us how many times to do that. The exponent is written as a small number above and to the right of the base (the number which is being multiplied by itself).

$$3^2 = 3 \times 3$$

$$5^3 = 5 \times 5 \times 5$$

We read numbers with exponents in the following way:

We read  $12^2$  as “12 to the second power,” or “12 squared.”

We read  $25^3$  as “25 to the third power,” or “25 cubed.”

We read  $8^4$  as “8 to the fourth power.”

We read  $3^5$  as “3 to the fifth power.”

**Example:** What is the value of  $23^3$ ?

$$23^3 =$$

$$23 \times 23 \times 23 =$$

$$529 \times 23 = 12,167$$

Final answer:  $23^3 = 12,167$

## Order of Operations

There are four primary operations in math: addition, subtraction, multiplication, and division. You might also find elements such as parentheses and exponents in a mathematical equation. When a math problem contains several operations, it's important to complete each one in the correct order, or we will get an incorrect answer.

The *order of operations* provides a clear set of rules to follow when you're solving problems that have several different operations. Here is the order in which operations are done in any equation:

- Parentheses
- Exponents
- Multiplication and Division
- Addition and Subtraction

**Step 1: Parentheses.** If a problem contains an operation in parentheses, we always complete this operation first, no matter what else is in the equation:

$$5 + (3 \times 5) - 17 + 4^2 + 10 \div 5 \times 2 =$$

$$5 + 15 - 17 + 4^2 + 10 \div 5 \times 2 =$$

In this equation, there is one operation in parentheses:  $3 \times 5$ . First we solve that and replace the parentheses with the result.

**Step 2: Exponents.** After taking care of any operations in parentheses, we look for exponents, and then solve them, replacing the base and exponent with the result:

$$5 + 15 - 17 + 4^2 + 10 \div 5 \times 2 =$$

$$5 + 15 - 17 + 16 + 10 \div 5 \times 2 =$$

Since  $4^2$  means  $4 \times 4$ , which is 16, we replaced  $4^2$  with 16.

**Step 3: Multiplication and Division.** Once the elements involving parentheses and exponents are completed, we can perform any multiplication and division processes in the order they appear in the equation, from left to right:

$$5 + 15 - 17 + 16 + 10 \div 5 \times 2 =$$

$$5 + 15 - 17 + 16 + 2 \times 2 =$$

Since  $10 \div 5$  came first in the equation, we perform that process first, which results in 2. Now we can move on and take care of the multiplication process, replacing  $2 \times 2$  with the result (4):

$$5 + 15 - 17 + 16 + 2 \times 2 =$$

$$5 + 15 - 17 + 16 + 4 =$$

**Step 4: Addition and Subtraction.** Now all that's left are addition and subtraction operations, which also get done in the order they appear, from left to right:

$$5 + 15 - 17 + 16 + 4 =$$

$$20 - 17 + 16 + 4 =$$

$$3 + 16 + 4 =$$

$$19 + 4 = 23$$

It may seem very confusing at first, but just remember the order of operations and follow it step by step:

- Parentheses
- Exponents
- Multiplication and Division
- Addition and Subtraction

Remember, sometimes multiplication is indicated with parentheses, like this:

$$5(3) = 5 \times 3 = 15$$

When you see a number in parentheses without any operation inside the parentheses, you know it is treated as a multiplication element, and done at the same time you do the multiplication and division. Here's an example:

**Example:**  $7(3) - 20 + 4^2 + (7 - 2)$

**Step 1:** Parentheses. We first perform the operation in parentheses.

$$\begin{aligned} 7(3) - 20 + 4^2 + (7 - 2) \\ 7(3) - 20 + 4^2 + 5 \end{aligned}$$

You'll notice the (3) is still there—that's because it's not an operation in parentheses; it's a way to show multiplication.

**Step 2:** Exponents. Next we solve exponents.

$$\begin{aligned} 7(3) - 20 + 4^2 + 5 \\ 7(3) - 20 + 16 + 5 \end{aligned}$$

**Step 3:** Multiplication and Division. Once parentheses and exponents are taken care of, we solve multiplication and division from left to right. There is just one multiplication process in this problem.

$$\begin{aligned} 7(3) - 20 + 16 + 5 \\ 21 - 20 + 16 + 5 \end{aligned}$$

**Step 4:** Addition and Subtraction. Finally, we can perform the addition and subtraction, from left to right.

$$\begin{aligned} 21 - 20 + 16 + 5 = \\ 1 + 16 + 5 = \\ 17 + 5 = 22 \end{aligned}$$

Let's look at one more example:

**Example:**  $16 + 9 \div (7 - 4) - 2 \cdot 3^2$

**Step 1:** Parentheses. We first perform the operation in parentheses.

$$\begin{aligned} 16 + 9 \div (7 - 4) - 2 \cdot 3^2 &= \\ 16 + 9 \div 3 - 2 \cdot 3^2 &= \end{aligned}$$

**Step 2:** Exponents. Next we solve exponents.

$$\begin{aligned} 16 + 9 \div 3 - 2 \cdot 3^2 &= \\ 16 + 9 \div 3 - 2 \cdot 9 &= \end{aligned}$$

**Step 3:** Multiplication and Division. Perform multiplication and division operations, from left to right.

$$\begin{aligned} 16 + 9 \div 3 - 2 \cdot 9 &= \\ 16 + 3 - 2 \cdot 9 &= \\ 16 + 3 - 18 &= \end{aligned}$$

**Step 4:** Addition and Subtraction. Finally, perform the addition and subtraction, from left to right.

$$\begin{aligned} 16 + 3 - 18 &= \\ 19 - 18 &= 1 \end{aligned}$$

### PEMDAS: The Order of Operations

In the order of operations, parentheses always come first, exponents always come second, and then multiplication and division are done in the order they appear (left to right), and finally addition and subtraction are completed from left to right.

If you find it hard to remember the order for all of these operations, you're not alone. Many mathematicians through the years have found it difficult to remember the correct order of operations. As a result, they have devised ways to remember this order, using the first letter of the operations in the proper order, as follows:

- **P**arentheses
- **E**xponents
- **M**ultiplication and **D**ivision
- **A**ddition and **S**ubtraction



The first letters of the operations in the proper order are P, E, M, D, A, and S. If you put them all together they form the word PEMDAS. This is what's called an *acronym*—a word that's created from the first letters of several words. An acronym can be very useful for remembering information, for you only have to remember the acronym and you have a clue to the words that make up each of the letters.

A phrase or sentence can also help you remember the order of operations:

**Please Excuse My Dear Aunt Sally**

The first letter of each of the words in this phrase are P, E, M, D, A, and S, so once again you're using these letters to remind you of the order of operations. This phrase can remind you of a lovable old aunt who is constantly doing silly things, so you're always asking others to please excuse her behavior.

Whatever way you use to remember the letters P, E, M, D, A, and S, repeat it to yourself when you're solving an expression that uses all of the operations, and it will help you complete each operation in the proper order.

### New Skills Practice

Complete the following worksheets in your math workbook:

- Lesson 1 New Skills Practice: Mean, Median, Mode, and Range; Exponents; and Order of Operations
- Lesson 1 Test

Remember to show all your work. Check your answers for the New Skills Practice and circle any incorrect answers before reworking these problems. Ask for help or use the additional practice worksheets if you need to.

Once you understand the material, complete the lesson test. Your parent will check your answers for the test and have you redo any incorrect problems.

### FOR ENROLLED FAMILIES

You will be sending work to your Oak Meadow teacher after every two lessons. Please check the answers for the lesson 1 test using the answer key in the appendix of the math workbook. Circle any incorrect problems. Score the test, and write the number correct over the total number at the top of the page. For instance, if there are 25 problems in the test and your student gets two wrong, you would write  $\frac{23}{25}$  at the top. Have your student redo any incorrect problems. Encourage your child to talk through the problem aloud so you can see where the error occurred and help your child fix it.

**All math work must be checked and corrected** so that your student learns how to perform each skill accurately and consistently. Students should check the answers on all worksheets themselves (anything other than tests), and make corrections. These practice worksheets won't be sent to your teacher, but completing them is an important element of this course.

Once this lesson is complete, move on to lesson 2. Feel free to contact your teacher if you have any questions about the assignments or the learning process.

# Lesson 2 Lowest Common Denominator in Fractions and Mixed Numbers

## Mental Math

Many of these mental math games are very quick. Repeat each game several times so that you are warming up your brain for 2–5 minutes.

**Version 1:** You'll need a pair of dice for this game. Roll the dice and create a proper fraction from the two numbers (a proper fraction has the smaller number on top). For instance, if you roll a 5 and a 3, the proper fraction will be  $\frac{3}{5}$ . Now add that fraction to itself:  $\frac{3}{5} + \frac{3}{5} = \frac{6}{5}$ . Finally reduce the fraction to its lowest terms:  $\frac{6}{5} = 1\frac{1}{5}$ . (Remember, you are doing all the calculations in your head.)

**Version 2:** Repeat the game above, but this time you will create a proper fraction from the two numbers, and then remember it while you roll the dice again and create a new fraction. Then you will multiply the two fractions in your head and reduce. For instance, if you roll a 2 and a 3, you will create a proper fraction ( $\frac{2}{3}$ ) and keep that fraction in your head while you roll the dice again. If you get a 6 and 5 on the second roll, you'll create a second proper fraction ( $\frac{5}{6}$ ), and then multiply the two:  $\frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$ . Remember to reduce your answer to lowest terms!

## Skills Check

Complete the following worksheet to practice some of the skills you have learned.

- Lesson 2 Skills Check

## New Skills

### Identifying Common Denominators and Lowest Common Denominator (LCD)

When we are adding or subtracting fractions whose denominators are the same, we simply add or subtract the numerators, and the denominator remains the same. But when the denominators are not the same, we have to identify a common denominator before we can add or subtract. A common denominator is a number that can be divided evenly by both denominators in the problem.

## ASSIGNMENT SUMMARY

- Play mental math games.
- Complete the Skills Check worksheet.
- Read New Skills instruction.
- Complete New Skills Practice.
- Complete Lesson 2 Test and Learning Checklist.

There are several ways to find a common denominator:

1. Use the largest denominator in the problem
2. Multiply the two denominators
3. Compare the multiples of both denominators and choose the lowest multiple that both fractions have in common

We'll use each approach in the following examples. Let's start with using the largest denominator in the problem.

**Example:**  $\frac{3}{4} + \frac{1}{12}$

**Step 1:** Look at the two denominators. In this example, we have denominators of 4 and 12. Since 4 goes into 12 evenly, we know we can use 12 as the common denominator.

**Step 2:** Convert  $\frac{3}{4}$  into an equivalent fraction with 12 as the denominator by asking, "How many times does 4 go into 12?" The answer is 3, so multiply the numerator by 3 to get the equivalent fraction of  $\frac{9}{12}$ .

**Step 3:** Complete the problem as usual, and reduce the answer to lowest terms.

$$\frac{9}{12} + \frac{1}{12} = \frac{10}{12} = \frac{5}{6}$$

You should always try this approach first because it's the easiest. However, it doesn't always work, so let's try the next approach: multiply the two denominators.

**Example:**  $\frac{6}{7} - \frac{2}{3}$

**Step 1:** In this problem we can't use the larger denominator because 3 won't divide evenly into 7, so we'll multiply the two denominators to find a common denominator:  $7 \times 3 = 21$ , so 21 is our new denominator.

**Step 2:** Convert each fraction into an equivalent fraction with a denominator of 21. First ask, "How many times does 7 go into 21?" The answer is 3 so we multiply the numerator of the first fraction by 3 to get the equivalent fraction of  $\frac{18}{21}$ . Then we do the same for the second fraction: "How many times does 3 go into 21?" The answer is 7 so we multiply the numerator of the second fraction by that to get the equivalent fraction of  $\frac{14}{21}$ .

**Step 3:** Complete the problem as usual, and reduce the answer to lowest terms.

$$\frac{18}{21} - \frac{14}{21} = \frac{4}{21}$$

Multiplying the two denominators will *always* give you a common denominator, but often this denominator will be quite large. To avoid this, always try to find the lowest common denominator (LCD).

**Example:**  $\frac{2}{3} + \frac{1}{4}$

**Step 1:** Look at the two denominators, and then identify the lowest common denominator by comparing the multiples of both denominators and choose the lowest multiple that both fractions have in common.

Multiples of 3:      3      6      9      **12**      15

Multiples of 4:      4      8      **12**      16      20

We can see that 12 is the lowest common denominator for the two denominators in this problem.

**Step 2:** Convert each fraction into an equivalent fraction using the LCD.

$$\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12}$$

**Step 3:** Complete the problem and reduce if necessary.

### LCDs in Mixed Numbers

You can follow these same procedures to find the lowest common denominator when you are adding or subtracting mixed numbers.

**Example:**  $3\frac{1}{4} + 7\frac{5}{6}$

**Step 1:** Find the lowest common denominator for the fractions. The lowest common denominator is 12.

**Step 2:** Convert the fractions to the common denominator.

$$\begin{array}{r} 3\frac{1}{4} = 3\frac{3}{12} \\ + 7\frac{5}{6} = 7\frac{10}{12} \\ \hline \end{array}$$

**Step 3:** Complete the problem. When you add mixed numbers using a common denominator, you often end up with a mixed number with an improper fraction. If so, reduce as usual.

$$\begin{array}{r}
 3\frac{1}{4} = 3\frac{3}{12} \\
 + 7\frac{5}{6} = 7\frac{10}{12} \\
 \hline
 10\frac{13}{12} = 11\frac{1}{12}
 \end{array}$$

### Borrowing with Mixed Numbers and LCDs

Sometimes the top fraction in a mixed number may not be large enough to subtract the bottom fraction. When this happens, just regroup (borrow) from the whole number.

**Example:**  $5\frac{1}{5} - 3\frac{3}{4}$

**Step 1:** Find the lowest common denominator. In this case, it is 20.

**Step 2:** Convert the fractions as usual.

$$\begin{array}{r}
 5\frac{1}{5} = 5\frac{4}{20} \\
 - 3\frac{3}{4} = 3\frac{15}{20} \\
 \hline
 \end{array}$$

**Step 3:** Since we can't subtract 15 from 4, we borrow 1 from the 5. We convert the 1 to an equivalent fraction using the common denominator:  $\frac{20}{20}$ . Now we can add that to  $\frac{4}{20}$ , and subtract as usual:

$$\begin{array}{r}
 5\frac{1}{5} = 5\frac{4}{20} = 4\frac{24}{20} \\
 - 3\frac{3}{4} = 3\frac{15}{20} = 3\frac{15}{20} \\
 \hline
 1\frac{9}{20}
 \end{array}$$

Sometimes, the value of the fractions in the mixed number may be the same, and the fraction in the answer will equal 0. In this case, since any fraction with 0 in the numerator equals 0, you can delete the fraction and just keep the whole number, as in the following example:

**Example:**  $15\frac{7}{8} - 12\frac{14}{16}$

$$\begin{array}{r}
 15\frac{7}{8} = 15\frac{14}{16} \\
 - 12\frac{14}{16} = 12\frac{14}{16} \\
 \hline
 3\frac{0}{16} = 3
 \end{array}$$

Always remember to reduce fractions in answers to lowest terms.

### New Skills Practice

Complete the following worksheets in your math workbook, showing all your work:

- Lesson 2 New Skills Practice: Lowest Common Denominator in Fractions and Mixed Numbers
- Lesson 2 Test

Check your answers for the New Skills Practice and circle any incorrect answers before reworking these problems. Use the additional practice worksheets if you need extra time to work on a skill.

Once you understand the material, complete the lesson test. Your parent will check your answers for the test and have you redo any incorrect problems.

### FOR ENROLLED FAMILIES

After your student completes the Skills Check and New Skills Practice for this lesson and the Lesson 2 Test (and makes any necessary corrections), please have your student complete the Lesson 2 Assessment Test. This is found in the math workbook.

Make sure the skills worksheets and the lesson 2 test have been corrected and your student has fixed any errors BEFORE taking the Assessment Test. All lesson tests should be scored (by you) and corrected (by your student) before being submitted to the teacher along with the Assessment Test. If you have any questions about this, please let your teacher know.

At the end of this lesson, submit the following three items to your Oak Meadow teacher:

- Lesson 1 Test
- Lesson 2 Test
- Lesson 2 Assessment Test

Do not include any of the practice worksheets (Skills Check, New Skills Practice, or extra practice worksheets).

Please include any additional notes about the lesson work or anything you'd like your teacher to know. Feel free to include questions—your teacher is eager to help.

If you have any questions about what to send or how to send it, please refer to your Parent Handbook and your teacher's welcome letter. Your teacher will respond to each submission of student work with comments and individualized guidance. In the meantime, have your student proceed to lesson 3 and continue working.



## Lesson



# Dividing Decimals; Factors and Prime Numbers

### Mental Math

As you perform your mental math, if you ever want to check your answer on paper, please do. It's not necessary, though—even if you get the answer wrong, just the act of doing calculations in your head will strengthen your math skills, even if you make a mistake now and then.

**Version 1:** Ask someone to give you a two-digit number.

Remember this number, and then reverse the digits to create a second number. Add the two numbers together. For instance, if the original number is 83, you add  $83 + 38 = 121$ . Do this several times. See if you notice a pattern. Then ask for a three-digit number and do the same (for instance,  $168 + 861$ ). Solve several three-digit problems. Feel free to challenge yourself with four-digit numbers, too!

**Version 2:** Ask someone for a two-digit number. Repeat this number aloud. Then in your head, create a new number by using the same digits again to form a four-digit number. Say this number aloud, and then add the same two-digits again to create a six-digit number, and say this number aloud. See if you can go up to a ten-digit number (billions place) or further. For instance, if the original number is 98, then the four-digit number is 9,898 (nine thousand, eight hundred ninety-eight); the six-digit number is 989,898 (nine hundred eighty-nine thousand, eight hundred ninety-eight); and so on.

### Skills Check

Complete the following worksheet to practice some of the skills you have learned.

- Lesson 6 Skills Check

### New Skills

#### Dividing Decimals by Whole Numbers

When dividing decimals, the process is the same as dividing whole numbers, except we have to account for the decimal point. The only difference is an adjustment we make to put the decimal point in the correct place.

### ASSIGNMENT SUMMARY

- Play mental math games.
- Complete the Skills Check worksheet.
- Read New Skills instruction.
- Complete New Skills Practice.
- Complete Lesson 6 Test and Learning Checklist.

**Example:**  $42.93 \div 3$

**Step 1:** Rewrite the problem using the division bracket. Remember that the number being divided (which goes inside the bracket) is the dividend; the number doing the dividing (which goes outside the bracket) is the divisor; and the answer is the quotient.

$$3 \overline{)42.93}$$

**Step 2:** Divide as usual, keeping the digits in the correct columns. Ignore the decimal point for now.

**Step 3:** To create the final answer, place the decimal point in the answer directly above where it is in the dividend.

$$\begin{array}{r} 14.31 \\ 3 \overline{)42.93} \\ \underline{3} \phantom{00} \\ 12 \phantom{00} \\ \underline{12} \phantom{00} \\ 09 \phantom{00} \\ \underline{9} \phantom{00} \\ 03 \phantom{00} \\ \underline{3} \phantom{00} \\ 0 \end{array}$$

You use this same process no matter how many decimal places there are in the dividend:

### Dividing with Dividends Less Than 1

When you divide a whole number into a dividend that is less than one, you have to pay attention to where the zeros are, and make sure the zeros are placed correctly in the quotient.

**Example:**  $0.25 \div 5$

$$5 \overline{)0.25}$$

**Step 1:** Treat the zero just like any other number. Say to yourself, “How many times does 5 go into 0?” Since 5 doesn’t go into 0, the answer is 0, so you write that in the quotient, then multiply, subtract, and bring down the next digit (the 2) as usual. Repeat the long division process of divide, multiply, subtract, and bring down until you have solved the problem.

**Step 2:** Place the decimal point in the answer so that it directly lines up with the decimal point in the dividend.

$$\begin{array}{r} 0.05 \\ 5 \overline{)0.25} \\ \underline{0} \\ 02 \\ \underline{0} \\ 25 \\ \underline{25} \\ 0 \end{array}$$

### Dividing Decimals with Remainders

When you have a remainder in a decimal division problem, you continue to add zeros to the end of the dividend until there is no remainder. Adding zeros to the end of the dividend doesn't change the value of the dividend, it just renames it, which allows us to complete the problem.

**Example:**  $8 \overline{)7.31}$

**Step 1:** Divide as usual, keeping the digits in the correct columns. When you end up with a remainder (3), add a zero to the dividend, bring it down, and continue dividing as usual. Continue dividing, adding zeros to the dividend and bringing them down until the answer comes out evenly and there is no remainder left. When you finish dividing, bring the decimal point directly up into the quotient for the final answer.

$$\begin{array}{r} 0.91375 \\ 8 \overline{)7.31000} \\ \underline{0} \\ 73 \\ \underline{72} \\ 11 \\ \underline{8} \\ 30 \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

## Rounding Decimals

Some decimal problems end up with answers that involve many decimal places. Since an answer in hundredths (two decimal places) is sufficient for most problems, the best solution is to round off the answer to two decimal places. In order to do that, you have to solve the problem through the thousandths place, and if there is still a remainder, follow the basic rules of rounding:

1. **If the digit in the thousandths place is 5 or greater, drop it and increase the digit in the hundredths place by 1**
2. **If the digit in the thousandths place is less than 5, drop it and keep the digit in the hundredths place as it is.**

**Examples of rounding decimals to the hundredths place:**

3.486 rounds to 3.49

0.0231 rounds to 0.02

12.9057 rounds to 12.91

72.302 rounds to 72.3

## Repeating Decimals

Sometimes you'll encounter a problem that results in a repeating decimal, decimal numbers that continue to repeat in a certain pattern, no matter how many zeros you add to the dividend. Here is an example:

**Example:**  $6 \overline{)0.2}$

You begin to solve the problem as usual, until you quickly realize the problem will continue to result in a remainder of 2 no matter how many zeros you bring down. This will result in a repeat of the number 3 in the quotient.

If you see that this is what the remainder will continue to be, you can either round off the answer or you can place a bar above two repeating digits to indicate that these digits will continue to repeat indefinitely (drop any extra repeating digits after the two barred digits). Either solution—rounding off or using a bar to indicate repeating decimals—is acceptable.

$$\begin{array}{r}
 \phantom{0.}033 \\
 6 \overline{)0.200} \\
 \underline{0} \phantom{00} \\
 02 \phantom{0} \\
 \underline{0} \phantom{0} \\
 20 \\
 \underline{18} \\
 20 \\
 \underline{18} \\
 2
 \end{array}$$

### Dividing Decimals by Decimals

When dividing using two decimal fractions, the division process is the same as usual, but the placement of the decimal point is different. Look at the following example:

**Example:**  $1.2 \overline{)20.46}$

- Step 1:** Move the decimal point in the divisor to the right until it is at the end of the number. That means instead of the divisor being 1.2, it's now 12. Since the divisor is now a whole number, it no longer needs a decimal point.
- Step 2:** Next, move the decimal point in the dividend the same number of spaces to the right that you moved it in the divisor. Since you moved it one place to the right in the divisor, you'll move it one place to the right in the dividend. That means the dividend will change from 20.46 to 204.6.

$$12 \overline{)204.6}$$

- Step 3:** Divide as usual, and then bring the decimal directly up into the quotient.

$$\begin{array}{r}
 \phantom{12.}05 \\
 12 \overline{)204.60} \\
 \underline{12} \phantom{00} \\
 84 \phantom{0} \\
 \underline{84} \phantom{0} \\
 06 \\
 \underline{0} \\
 60 \\
 \underline{60} \\
 0
 \end{array}$$

Because we've moved the decimal point the same number of places in both the divisor and the dividend, we haven't changed the relationship between the two numbers. When we move the decimal point one place to the right in any number, we're actually multiplying the number by 10. Since we're multiplying both the divisor and the dividend by the same amount, then the relative value of the two numbers remains the same. Always remember to move the decimal points in both the divisor and the dividend by the same number of places.

Sometimes we'll see a problem where both the divisor and the dividend are less than one. You might see a zero in front of the decimal but usually decimals less than one are written without the "leading" zeros in front; either way, the value is the same.

**Example:**  $.03 \overline{) .46}$

**Step 1:** Turn the divisor into a whole number by moving the decimal in the divisor to the right until it's at the end of the number (.03 becomes 3). Next, move the decimal point the same number of places in the dividend (.46 becomes 46).

$$3 \overline{) 46}$$

**Step 2:** Divide as usual, and bring the decimal point up into the quotient. In this problem, you'll quickly discover that there is a repeating decimal, so you can either round the answer to 15.33 or place a bar over the .33 to show it is a repeating decimal.

$$\begin{array}{r} 15.\overline{33} \\ 3 \overline{) 46.00} \\ \underline{3} \phantom{00} \\ 16 \phantom{00} \\ \underline{15} \phantom{00} \\ 10 \phantom{00} \\ \underline{9} \phantom{00} \\ 10 \phantom{00} \\ \underline{9} \phantom{00} \\ 0 \end{array}$$

### Dividing Whole Numbers by Decimals

Finally, let's look at dividing whole numbers by decimals. Keep in mind that everything to the left of a decimal point is a whole number, and everything to the right of a decimal is a fraction. Whole numbers don't need a decimal point, but we can add one and place as many zeros after it as we like without changing the value of the whole number. All of these numbers have the same value:

$$32 \quad 32.0 \quad 32.00 \quad 32.000$$

When we divide a whole number by a fraction, simply add a decimal point and zeros in order to help us solve the problem:

**Example:**  $.4 \overline{)29}$

**Step 1:** Change the whole number in the dividend into a decimal fraction by adding a decimal and zeros. You may want to add more zeros later, but start with two zeros.

$$.4 \overline{)29.0}$$

**Step 2:** Continue as you would to divide a decimal into a decimal. Move the decimal in the divisor and the dividend the same number of spaces:

$$4 \overline{)290.}$$

**Step 3:** Divide as usual. Remember that when you are dividing with decimals, you continue to add zeros until the problem comes out evenly with no remainders. Finally, place the decimal point in the answer, directly above where it now is in the dividend.

$$\begin{array}{r} 72.5 \\ 4 \overline{)290.0} \\ \underline{28} \phantom{0} \\ 10 \phantom{0} \\ \underline{8} \phantom{0} \\ 20 \phantom{0} \\ \underline{20} \\ 0 \end{array}$$

### Factors of Whole Numbers

Every whole number greater than 1 has at least two **factors**—1 and the number itself—and many whole numbers have more than two factors. Factors are all the whole numbers that can divide evenly into a number.

An easy way to determine the factors of a number, after writing down 1 and the number, is to start with 2 and work your way up through the numbers. If the number is even, 2 and another number will be factors. For instance, if we are looking for the factors of 12, we know  $2 \times 6 = 12$ , so 2 and 6 are factors. Next, see if 3 goes into the number evenly; if so, write down 3 and the number it is paired with. We know  $3 \times 4 = 12$  so 3 and 4 are factors. Continue working up through the numbers (does 4 go into the number evenly? Does 5?). In this way, you can quickly determine the factors of a number.

**Example:** What are the factors of 24?

Does 1 go into 24? Yes, so 1 and 24 are factors.

Does 2 go into 24? Yes, so 2 and 12 are factors.

Does 3 go into 24? Yes, so 3 and 8 are factors.

Does 4 go into 24? Yes, so 4 and 6 are factors.

Does 5 go into 24. No.

Does 6 go into 24? Yes, but we've already written that down. Once you get to a repeat factor, you know we've found all the factors.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

**Example:** Write the factors of 60.

Does 1 go into 60? Yes, so 1 and 60 are factors.

Does 2 go into 60? Yes, so 2 and 30 are factors.

Does 3 go into 60? Yes, so 3 and 20 are factors.

Does 4 go into 60? Yes, so 4 and 15 are factors.

Does 5 go into 60. Yes, so 5 and 12 are factors.

Does 6 go into 60? Yes, so 6 and 10 are factors.

Does 7 go into 60? No.

Does 8 go into 60? No.

Does 9 go into 60? No.

Does 10 go into 60? Yes, but we've already written that down so we know we're done.

The factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.

## Prime Numbers

While some numbers have several factors, others have just two factors: 1 and the number itself. These numbers are called *prime numbers*. A prime number cannot be divided evenly by any number except itself and 1.

**Example:** What are the factors of 13?

Applying the same technique we used above, see how many numbers will divide evenly into 13.

Does 1 go into 13? Yes, so 1 and 13 are factors.

Does 2 go into 13? No.

Does 3 go into 13? No.



Does 4 go into 13? No.

Does 5 go into 13. No.

Does 6 go into 13? No.

Does 7 go into 13? No.

Once you get past the halfway point, you don't have to go any further because any factors would have already been discovered—we already know 2 doesn't divide evenly into the number.

Since 13 has exactly two factors (1 and 13), it is a prime number.

### **New Skills Practice**

Complete the following worksheets in your math workbook:

- Lesson 6 New Skills Practice: Dividing Decimals; Factors and Prime Numbers
- Lesson 6 Test

Show all your work and check your answers, reworking any incorrect problems.

### **FOR ENROLLED FAMILIES**

At the end of this lesson, submit the following three items to your Oak Meadow teacher:

- Lesson 5 Test
- Lesson 6 Test
- Lesson 6 Assessment Test

Make sure the two lesson tests have been graded (by you) and then corrected (by your child). Do not include any of the practice worksheets with your submission.

## Lesson



# Percentages, Simple and Compound Interest

### Mental Math

**Version 1:** You'll need a pair of dice for this game. Roll the dice and form a number. For instance, if you roll a 2 and a 3, you can form either 23 or 32. Think up all the factors of that number. Take note of any numbers that are prime. Do this several times.

**Version 2:** You'll need three dice for this game. Roll two of the dice and form a number with the larger number first. Then roll the third die and divide the original number by the number on the third die. If you have a remainder, express it in the form of a fraction (by putting the remainder over the divisor). For instance, if you first roll a 6 and a 2, you form the number 62. Then if you roll the third die and get a 3, you use it to divide:  $62 \div 3 = 20\frac{2}{3}$ . Next, reverse the digits on your original roll and divide again:  $26 \div 3 = 8\frac{2}{3}$ . Do this several times.

### Skills Check

Complete the following worksheet to practice some of the skills you have learned.

- Lesson 7 Skills Check

### New Skills

#### Multiplying and Dividing Decimals by 10, 100, and 1,000

When you multiply or divide any number by 10, 100, or 1,000 (or any whole number formed by 1 with zeros after it), you simply move the decimal point to the left when dividing (which creates a smaller number) or to the right when multiplying (which creates a larger number). The number of spaces we move the decimal point is determined by the number of zeros in the multiplier or the divisor. Let's look at how this works with multiplying first.

### ASSIGNMENT SUMMARY

- Play mental math games.
- Complete the Skills Check worksheet.
- Read New Skills instruction.
- Complete New Skills Practice.
- Complete Lesson 7 Test and Learning Checklist.

**Example:**  $12 \times 10$ 

There is one zero in the multiplier (10). Since 12 can also be expressed as 12.0, we move the decimal point one place to the right.

$$12 \times 10 = 120$$

**Example:**  $1,532 \times 1,000$ 

There are three zeros in the multiplier (1,000). Since 1,532 can also be expressed as 1,532.000, we move the decimal point three places to the right.

$$1,532 \times 1,000 = 1,532,000$$

This process works the same way when multiplying decimals.

**Example:**  $12.25 \times 10$ 

There is one zero in the multiplier (10), so we move the decimal point one place to the right.

$$12.25 \times 10 = 122.5$$

**Example:**  $32.75 \times 100$ 

There are two zeros in the multiplier (100), so we move the decimal point two places to the right.

$$32.75 \times 100 = 3,275$$

**Example:**  $.31 \times 10,000$ 

There are four zeros in the multiplier, so we move the decimal point four places to the right.

$$.31 \times 10,000 = 3,100$$

Now let's apply this same process to division. Since dividing results in smaller numbers, we need to move the decimal point to the left instead of the right.

**Example:**  $75 \div 10$ 

Remember, all whole numbers can be expressed with a decimal point to the right of the digits—we don't normally place it there unless there is a decimal fraction, but it is assumed to be there at all times. Since there is one zero in the divisor (10), we move the decimal point one place to the left.

$$75 \div 10 = 7.5$$

**Example:**  $23 \div 100$ 

Since there are two zeros in the divisor (100), we move the decimal point two places to the left.

$$23 \div 100 = .23$$

**Example:**  $87 \div 1,000$ 

Since there are three zeros in the divisor (1,000), we move the decimal point three places to the left. Sometimes this means adding zeros as place holders.

$$87 \div 1,000 = .087$$

**Example:**  $5 \div 10,000$ 

Since there are four zeros in the divisor, we move the decimal point four places to the left, adding zeros as necessary.

$$5 \div 10,000 = .0005$$

**Calculating Percentages**

Decimals are one way to express a fraction. A **percentage** is another way of expressing a fraction. The word *percent* means “of a hundred,” meaning the whole is divided into 100 parts.

$$20\% = 20 \text{ parts of } 100 \text{ or } \frac{20}{100}$$

$$85\% = 85 \text{ parts of } 100 \text{ or } \frac{85}{100}$$

To convert a percentage to a decimal, we move the decimal point two places to the left, dividing it by 100. This works because a percentage is based upon 100.

**Example:** Write 25% as a decimal.

First, remove the percent sign and then move the decimal point two places to the left.

$$25\% = .25$$

**Example:** Write 9% as a decimal.

Remove the percent sign and move the decimal point two places to the left, adding a zero if necessary.

$$9\% = .09$$

The zero acts as a placeholder, showing us the value of the decimal is  $\frac{9}{100}$ . If we didn't add the zero, but instead wrote .9, we would be showing  $\frac{9}{10}$ , not  $\frac{9}{100}$ .

Sometimes we need to calculate the percentage of a certain number. To do this, we change the percent to a decimal and then multiply, as seen in the following examples.

**Example:** How much is 50% of 45?

**Step 1:** Change 50% into a decimal by moving the decimal point two places to the left:

$$50\% = .5$$

**Step 2:** Multiply the number by the decimal. Put the number with the most digits on top—that just makes it easier.

$$\begin{array}{r} 45 \\ \times .5 \\ \hline 22.5 \end{array}$$

This shows us that 50% of 45 is 22.5.

**Example:** What is 75% of 400?

**Step 1:** Convert the percentage into a decimal:  $75\% = .75$ .

**Step 2:** Multiply the number by the decimal.

$$\begin{array}{r} 400 \\ \times .75 \\ \hline 2000 \\ \underline{2800} \\ 300 \end{array}$$

This shows us that 75% of 400 is 300.

**Example:** What is 3% of 120?

**Step 1:** Convert the percentage into a decimal:  $3\% = .03$ . Remember to be careful about adding the leading zero when converting a single digit percentage to a decimal.

**Step 2:** Multiply the number by the decimal.

$$\begin{array}{r} 120 \\ \times .03 \\ \hline 3.60 \end{array}$$

This shows us that 3% of 120 is 3.6.

We'll often see percentage questions as part of a word problem, like the following example.

**Example:** You-Pick Apple Orchards sold 175 bushels of apples in September, with 95% of the sales going to families who picked apples with their children. How many bushels were picked by families with children in September?

This problem is really asking “What is 95% of 175?” So we follow the steps to find out.

**Step 1:** Convert the percentage into a decimal:  $95\% = .95$ .

**Step 2:** Multiply the number by the decimal.

$$\begin{array}{r} 175 \\ \times .95 \\ \hline 875 \\ \underline{1575} \\ 166.25 \end{array}$$

There were 166.25 bushels of apples picked by families with children in September.

Percentages aren't always expressed as whole numbers, but that doesn't change the process of how we deal with them.

**Example:** On average, 62.5% of book sales at Reader's Haven come from fiction novels. If the store sold 28 books yesterday, how many of those books were likely to be fiction novels?

This question is really asking, “What is 62.5% of 28?”

**Step 1:** Convert 62.5% into a decimal by moving the decimal point two places to the left:  $.625$ .

**Step 2:** Multiply as usual. Since the decimal has more digits, we put it on top.

$$\begin{array}{r} .625 \\ \times 28 \\ \hline 5000 \\ \underline{1250} \\ 17.5 \end{array}$$

Of all the books sold yesterday, 17.5 were likely to be fiction novels.

(Are you wondering how a store can sell 17.5 books? How can someone sell half a book? The answer is they can't, of course, but we're looking at an average amount over time. This means that the store probably sold 17 or 18 fiction books in one day, based on the average of 62.5% of the total book sales. Mathematically, however, the answer comes out to 17.5.)

**Example:** Jessie spent \$32.54 on new shoes and a sales tax of 7.5% was added to this amount. How much was the sales tax? How much was the total bill?

This question is asking, “How much is 7.5% of \$32.54?”

**Step 1:** Convert the percentage into a decimal:  $.075$ .

**Step 2:** Multiply as usual. This time you are multiplying two decimals.

$$\begin{array}{r}
 \$32.54 \\
 \times \quad .075 \\
 \hline
 16270 \\
 22778 \\
 \hline
 \$2.4405
 \end{array}$$

**Step 3:** Since we are working with money, we have to round the answer to two places, so \$2.4405 becomes \$2.44. The sales tax came to \$2.44.

**Step 4:** Add the sales tax to the cost of the shoes to get the total:

$$\begin{array}{r}
 \$32.54 \\
 + \quad 2.44 \\
 \hline
 \$34.98
 \end{array}$$

The total bill came to \$34.98.

## Interest and Principle

When money is borrowed, usually *interest* is charged; this is basically a fee for using the money. Whenever people borrow money to buy a car, a home, or make a purchase using a credit card, they are obligated to pay back not only the amount they borrowed, which is called the *principal*, but also the interest, which is calculated as a percentage of the principal.

There are two types of interest: simple and compound. We'll introduce *simple interest* first.

Simple interest is an amount calculated at a specific percentage rate for a specific period of time. For example, suppose someone borrows \$10,000 for one year at an interest rate of 12% per year. At the end of the year, the principal (\$10,000) must be repaid along with the accumulated interest (this is often called *accrued interest*). Since 12% of \$10,000 is \$1,200, at the end of the year, the person must pay \$11,200 (\$10,000 principal + \$1,200 interest).

Unless specified otherwise, interest is based upon one year. So if an interest rate is listed as 8%, it means 8% per year, even if the loan period is longer than that. Look at the following example:

**Example:** What is the simple interest on \$20,000 at 8% for three years?

First we calculate the interest for one year:

$$\$20,000 \times 8\% = \$20,000 \times .08 = \$1,600$$

Then we multiply that by three years:

$$\$1,600 \times 3 \text{ years} = \$4,800$$

The simple interest comes to \$4,800. This is the amount of money that will be paid in addition to repaying the original loan.

**Example:** Jonah loaned Michael \$15,000 for two years at 10% simple interest. How much will Michael owe Jonah at the end of two years?

At the end of two years, Michael will owe Jonah the principal plus the interest. We need to calculate the interest for two years and add that to the principal to determine the total amount due.

**Step 1:** Calculate the interest for one year:

$$\$15,000 \times 10\% = \$15,000 \times .10 = \$1,500$$

**Step 2:** Multiply that by 2 years to get the total interest due.

$$\$1,500 \times 2 \text{ years} = \$3,000$$

**Step 3:** Add the principal and the interest to get the total amount due.

$$\$15,000 + \$3,000 = \$18,000$$

At the end of 2 years, Michael will owe Jonah \$18,000.

### Compound Interest

Simple interest is calculated only on the original principal. Simple interest helps us to understand the concept of interest and principal, but it is rarely used except in small loans between individuals. Businesses, banks, and other financial institutions commonly use *compound interest*, which is calculated on the original principal plus whatever interest has been added each day, month, or year. When we borrow money, the compound interest is added to the amount we have to repay.

Banks usually compound interest on a daily basis, but to simplify the calculations in the examples that follow, we will compound the interest once per year (annually).

**Example:** If you borrowed \$10,000 at 10% interest compounded annually for 3 years, how much would you owe at the end of three years?

**Step 1:** Calculate the interest for the first year.

$$\text{Year 1: } \$10,000 \times 10\% = \$1,000$$

This is the interest for one year. Because it is compounded, we add this to the principal, and this amount becomes the *new principal* for the next year.

$$\$10,000 + 1,000 = \$11,000$$

This is the new principal for Year 2.

**Step 2:** Calculate the interest for the second year, based on the new principal.

$$\text{Year 2: } \$11,000 \times 10\% = \$1,100$$



Once again, we add this interest to the principal.

$$\$11,000 + 1,100 = \$12,100$$

This is the principal for Year 3.

**Step 3:** Repeat the process for the third year.

$$\text{Year 3: } \$12,100 \times 10\% = \$1,210$$

We add this to the principal.

$$\$12,100 + 1,210 = \$13,310$$

The value of \$10,000 compounded annually at 10% for three years is \$13,310.

The simple interest on this amount would have been \$1,000 per year for three years, or \$3,000. The compound interest, which increased each year, came to \$3,310. You can see that compound interest adds up more quickly than simple interest.

Compound interest isn't only used when borrowing money—it can also be earned. Many people put money aside to save, either in a savings account or in another type of account or investment that earns interest. When we put money in an interest-bearing account in a bank, we are actually loaning money to the bank. The bank uses our money to make investments that earn money for the bank. In exchange for the use of our money, the bank agrees to pay us a specified rate of interest, compounded on a regular basis.

**Example:** What is the value of \$50,000 compounded annually at 20% for 5 years?

**Step 1:** Calculate the interest for the first year.

$$\text{Year 1: } \$50,000 \times 20\% = \$10,000$$

Add this interest to the principal; the new principal is \$60,000.

**Step 2:** Calculate the interest for each year, adding the interest to the principal each year to calculate a new principle.

$$\text{Year 2: } \$60,000 \times 20\% = \$12,000$$

Add this interest to the principal; the new principal is \$72,000.

$$\text{Year 3: } \$72,000 \times 20\% = \$14,400$$

Add this interest to the principal; the new principal is \$86,400.

$$\text{Year 4: } \$86,400 \times 20\% = \$17,280$$

Add this interest to the principal; the new principal is \$103,680.

$$\text{Year 5: } \$103,680 \times 20\% = \$20,736$$

Add this interest to the principal; the new principal is \$124,416.

The value of \$50,000 compounded annually at 20% for 5 years would be \$124,416.

### New Skills Practice

Complete the following worksheets in your math workbook:

- Lesson 7 New Skills Practice: Percentages, Simple and Compound Interest
- Lesson 7 Test

Show all your work and check your answers, reworking any incorrect problems.