Math Connections

Coursebook

Oak Meadow

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Introduction

Welcome to Math Connections! This course will answer the common math question, "When will I ever use this?" The overarching theme of this course is problem-solving in the real world. The first chapter introduces problem-solving techniques that set the stage for the entire course. It is not always easy to solve problems and it is important that you do not give up when you don't immediately understand a problem or see how it will be solved. This course is designed to help you strengthen both your math skills and your ability to persevere through challenges.

You will be asked to explore historical topics to illustrate the challenges and processes that mathematicians—people just like you—worked through to arrive at the discoveries that we take for granted today. This will, hopefully, help you to appreciate the nature of problem-solving as a challenging and rewarding process even when a solution is difficult to achieve. As the famed inventor Thomas Edison is credited with saying, "Results? Why, man, I have gotten lots of results! If I find 10,000 ways something won't work, I haven't failed. I am not discouraged, because every wrong attempt discarded is often a step forward." When we think about "wrong attempts" as being a necessary step on the path to success, it empowers us to get back up and keep moving.

Some of the material in the textbook chapters will likely look familiar to you from Algebra I and Geometry, but this overlap is intentional. In Math Connections, we will review the basics so you have the opportunity to master concepts you may have missed in an earlier course and solidify your understanding of the underlying mathematical properties and structure. We will then dig more deeply, applying topics to real-world problems.

Other topics will probably be new to you, and may include ideas that you wouldn't typically think of as being "math." In this course, we will explore connections to other areas of math, as well as to various disciplines such as history, literature, science, art, and philosophy, in order to introduce you to the beauty and wonder that is mathematics. Try to approach unfamiliar topics with an open mind and a sense of curiosity.

How the Course Is Set Up

This two-semester course consists of 13 lessons, plus a midterm project and a final project. The first semester contains lessons 1 through 6 and the midterm project. The second semester contains lessons 7 through 13 and the final project. Each lesson corresponds to one of the 13 chapters in the textbook, Blitzer's *Math for Your World* (Pearson 2016).

Each lesson provides an estimate of how long it may take to complete, ranging from one to three weeks per lesson. Please note, however, that some lessons may take more or less time, depending on your experiences and abilities. The time estimate is a general guideline, so adjust your own pace as it fits for you.

In each lesson, you'll find the following types of assignments:

Mental Math

Mental math is any calculation that you do in your head. Mental math skills are important for completing tasks and making decisions in everyday life. Mental math also involves *how* to think mathematically to solve problems and understanding *why* certain steps are used. Throughout this course, we will do mental math as a warm-up activity for each day you work on math. Within each lesson, there will be a new mental math technique, strategy, or game for you to add to your "bag of tricks"—your toolkit of mental math skills. The more mental math practice you do, the better you will get at making mental calculations and the better you will understand the *how* and *why* of what you do. The key is to get into the habit of regularly doing the mental math activities. As with learning music, it is better to practice a single short exercise per day than to do them all in a longer session once per week. The mental math activities are designed to take only a few minutes each day.

Readings and Exercise Sets

Each textbook chapter is divided into sections that include Checkpoints and an Exercise Set. Read one section at a time, completing the Checkpoint problems along the way. At the end of each section, complete the Exercise Set before moving on to the next section. You will find answers in the back of the textbook—this lets you check your work and make corrections right away. Most Exercise Sets will include the following:

- **Concept and Vocabulary Exercises** can be done verbally with your home teacher.
- **Practice Exercises** provide a wide variety of problems. You will choose a selection of odd-numbered problems from each section to complete. (The answer key only includes answers to odd-numbered practice exercises, so it's important to make your selections from odd-numbered problems only.) This lets you customize the practice work to your own needs. You may only need to practice ten problems in one section, yet need to practice twenty or more problems in a section that you find more challenging. It is very important to make sure that you choose enough problems to ensure that you fully understand the concepts, and that you cover at least one of every kind of problem. It is your responsibility (with the help of your home teacher) to determine the problems you should complete.
- **Application Exercises** help you develop your problem-solving skills and see how the concepts in the chapter connect with real-world problems. In most lessons, you will be completing all odd-numbered Application Exercises.
- **Critical Thinking and Technology Exercises**, found at the end of some Exercise Sets, are optional and can be done verbally or in written form. You do not have to do the Group Exercises.

Math Journaling

Have you ever easily solved a problem but drawn a blank when you were asked to explain *how* you solved it? Being able to understand what you are doing in math and to clearly communicate your ideas are valuable and essential components of learning and doing mathematics. Each lesson in this course contains one or more journal assignments. Writing journal entries will help you develop critical thinking skills, synthesize and apply the concepts you have learned, reinforce your math vocabulary, and strengthen your ability to communicate your mathematical thinking.

Activities

At the end of each lesson, you will tie together everything you have learned in the lesson with an activity. Activities are deeper application problems that allow you to explore the lesson's concepts in a concrete or thought-provoking way. Some activities will have you connect what you learned to another discipline, such as art, history, English, or philosophy. Other activities will explore the work of a mathematician, make use of online tools and games, and require you to use research and problem-solving skills to gather information and make an informed decision.

Tests

Tests in this course are designed to be open book. You may refer to the textbook and your notes as needed. The Chapter Test at the end of each textbook chapter serves as the test for students who are completing the course independently. For students enrolled in Oak Meadow School, it serves as an optional—but recommended—practice test. Enrolled students are required to complete an alternate test which will be supplied by the school.

Midterm and Final Projects

Each semester contains a large-scale project due in the final week of the semester. The midterm project assignment is to write a creative biography on a mathematician of your choice. The final project assignment is to research and create a project on a topic of your choice that relates to math. Each project will be worked on in stages throughout the semester. Certain lessons contain Project Milestone assignments that will guide you through the project development process and keep you on track to complete the project by the end of the semester.

People in Mathematics

Each lesson contains a short biographical sketch of a mathematician whose work relates in some way to the chapter's topic. These individuals were selected because of their significant contributions to mathematics and, in some cases, because of their strong character in the face of adversity. They can serve as an inspiration to everyone facing challenges, reminding us to persevere and problem-solve.

Appendix

At the end of this coursebook you will find an appendix which includes original work guidelines, information on how to avoid plagiarism, and detailed instructions on finding reputable sources and citing sources correctly. Take a few minutes to look over this information so you can refer back to it as needed throughout the year. You will be expected to know this information and use it in your work.

Course Expectations and Tips

Here are some guidelines to help you get the most out of this course.

- Carefully read the information in the coursebook and textbook. The textbook was selected for its excellent readability and step by step example solutions. It's worth taking the time to thoroughly read the textbook assignments and follow along with the examples.
- Do not automatically skip material that looks familiar. Skipping over review topics may seem like a way to save time and energy, but mathematics is a subject that builds on itself, so it is necessary to periodically review concepts in order to keep them fresh and provide a solid foundation from which to learn the next layer of material. Taking the time to read through examples and work through problems, even those that contain ideas that you are already familiar with, will help you to better understand the underlying mathematical structure and ready your mind for new ideas.
- Do the Checkpoint problems after each example, and compare your answers to the ones in the back of the book. This will help you gauge whether you understood the problem. Tip: bookmark the solutions page you are on with a sticky note for quick and easy reference.
- Always check your Exercise Set solutions with the answers in the back of the textbook and make corrections as appropriate. This is a very important step in the independent learning process as it gives you an opportunity to assess your understanding of the course material as you learn it. Ask for help if you get stuck!
- **Show all of your steps for test problems.** Any step that cannot easily be done mentally must be written down in an organized and mathematically-valid format. If a problem does not require multiple steps, then it is wise to write a short explanation indicating how you arrived at your answer. In order to get into the habit of showing your steps on tests, you should practice showing your steps on your practice exercises, as well.
- Use tools such as a ruler, compass, and protractor as appropriate to make your graphs and diagrams neat and precise.
- Always check your answers for completeness and label answers with units.
- Do the mental math warm-up each day. Daily practice keeps ideas fresh in your mind, trains you to work with new ideas, and over time you will notice that you can make calculations faster and your overall math performance will be stronger. It's important to be consistent about your mental math practice. Don't skip these exercises!

- Write thoughtful, well-developed journal responses. You may choose to write journal responses by hand or type them out. While perfection is not expected, your responses should be clearly written and make sense. Check over your writing and make quick edits as necessary to ensure readability.

Use of Technology

A calculator will be needed for this course. If you plan to eventually take Algebra II, Advanced Math, or Calculus, a graphing calculator will be required for those courses. You are encouraged to invest in a graphing calculator (such as the TI-83 Plus or TI-84 Plus) early on so you can become familiar with its use. These calculators are frequently used in college math and science courses, as well, so you should get a lot of use out of this tool.

If you do not plan to take the courses listed above, then a scientific calculator will suffice. Please note: a simple four-function calculator will *not* be powerful enough for use in this course. A scientific calculator has buttons for trigonometric functions (sin, cos, and tan) and also has a key for exponents.

You are permitted to use your calculator on all assignments, with the exception of mental math exercises that are intended to be done mentally.

This course also makes use of Internet resources including websites, videos, and online games. You will need Internet access and a device with a browser capable of viewing and running these sites and applications. The course can be done without using the Internet, but accessing these online sources can enhance your learning experience. Enrolled students who do not have access to the Internet should contact their teacher to make alternate arrangements.

For Students Enrolled in Oak Meadow School

At the end of each lesson, you will see a section for enrolled students where you will be reminded of what to submit to your teacher. Here are some additional notes:

- The journal entries and activities are submitted to your teacher at the end of each lesson and will be included in your grade for the lesson.
- Exercise sets are not submitted to your teacher and will not be included in your grade for the lesson. However, working on exercise sets is a necessary step in the learning process.
- When you submit a Project Milestone assignment to your teacher, it is necessary to get approval before moving on to the next project milestone.
- **Always show your work.** Simply writing down the answer is not enough. You will be marked down if you do not show your work.

You will find a Lesson Test for each chapter in the testing packet that you download from the Oak Meadow Gateway. The Lesson Test is submitted to your teacher and will be included in your grade for the lesson. Take the Lesson Test only after you have fully reviewed and prepared for it. We recommend that you first take the textbook's Chapter Test and use it as a practice test to identify any lingering trouble spots. If you are still confused when making corrections, it is important that you ask for help from your home teacher or Oak Meadow teacher. It is always best to resolve trouble spots *before* taking the graded test.

Let the Journey Begin

Now that you have a good idea of what to expect, get ready to experience math in a new way. We hope you'll come to appreciate math and see it as an important tool in your life.

Lesson

1

Problem Solving and Critical Thinking

We will begin this course by exploring techniques for solving and thinking critically about problems—not just mathematical problems, but any type of problem we could encounter in our everyday lives. Then, using a four-step problem-solving process, you will practice looking for patterns and solving a variety of math problems and puzzles.

This lesson should take approximately two weeks to complete.

Learning Objectives

- Distinguish between and use inductive and deductive reasoning.
- Use estimation techniques to find approximate answers to problems.
- Explain the purpose and features of circle graphs, bar graphs, and line graphs.
- Apply estimation techniques to information presented on graphs.
- Estimate relationships between variables through mathematical modeling.
- Solve problems using the four-step problem solving process.
- Explore puzzles and games that involve problem-solving.

Why It Matters

While numbers are often the first thing that comes to mind when talking about math, the truth is that mathematics is also the study of several other ideas, including patterns. Patterns play a huge role not only in mathematics but in all disciplines, from the identification

ASSIGNMENT SUMMARY

- ☐ Mental Math Set A: The Sums Game
- ☐ Mental Math Set B: Rounding and Estimation Games
- ☐ Read Chapter 1 in textbook.
- ☐ Complete a selection of exercises for sections 1.1 through 1.3.
- ☐ Read Chapter 1 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: Math and You
- ☐ Math Journal B: The Four Step Problem Solving Method in Action
- ☐ Activity A: Modeling College Graduation Rate
- ☐ Activity B: Figurate Numbers and Pascal's Triangle
- ☐ Activity C: Logic Puzzles

> *A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on mind and character for a lifetime.*
>
> **George Pólya, *How to Solve It***

of themes in a novel or the exploration of motifs in a work of art or music to recognizing the similarity of repeated events in history or observing trends in the impact humans have on the environment. Patterns are everywhere, but we need to recognize them and clearly articulate their structure in order to show their significance.

Problem-solving is another aspect of mathematics that connects to many other areas of life. There are always problems to be solved, but doing so can be quite a challenge! Fortunately, there are tools that can help us with the problem-solving process. By practicing with these tools, we can train our brains to look for patterns and generate ideas for solutions. We can then apply these skills to solve problems in all areas of our lives and to help make the world a better place.

Mental Math Warms-Ups

Each lesson will contain at least one set of mental math activities. These exercises are meant to strengthen your mental math skills over time and to help you learn and discover new techniques to use in your everyday life. They are intended to be quick, so spend just three to five minutes practicing at the beginning of each math session.

You are encouraged to do your mental math exercises together with a partner to enhance learning and make the process more fun. Ask a parent, tutor, sibling or friend to be your partner. You can make it a collaborative effort where you make up and solve problems together with your partner, or you could create some friendly competition by challenging your partner to beat the clock or see who gets the most correct answers. Feel free to adapt the mental math exercises in any way that will make them more fun and engaging for you.

If you are unable to work with a partner, the mental math exercises can be adapted for solo practice by creating a short list of problems to challenge yourself with. You should write your problem list on a piece of paper. As you work, write down your answers, but **all calculations should be done in your head.** You can then use a calculator to verify your answers. To make things more interesting, you could challenge yourself to beat the clock or to get all answers correct.

Note for students enrolled in Oak Meadow School: Mental math activities are not submitted to your teacher. They will not appear on tests, and do not count toward your grade. However, it is expected that you will complete the mental math activities as they will strengthen your overall mathematical skills and understanding.

This lesson contains two sets of mental math warm-ups:

- Mental Math Set A: The Sums Game and Rounding
- Mental Math Set B: Estimation Games

Complete one set each week.

Mental Math Set A: The Sums Game

For the first week of mental math, you will be strengthening your mental addition, subtraction, and grouping skills through a fast-paced game. It is best to work with a partner, but you could also play the game on your own. Spend two to three minutes playing this warm-up game at the start of each math session to sharpen your mental math skills. Even better, play it any time you're bored (riding in the car or in line at the store, for example).

Day 1: The goal of the first Sums Game is to find pairs of numbers that add to 100. One person calls out a number between zero and one hundred and the other person must mentally figure out what number must be added to the first number to get to 100. For example, if your partner gives the number 64, you would answer 36 because $64 + 36 = 100$. Take turns calling out numbers for each other. As you get the hang of the game, try to answer more and more quickly. Afterward, discuss with your partner what strategies you both used to find the numbers. Did you use the same strategies? Perhaps your partner found a trick that you can try next time.

Days 2 and on: Continue to play the Sums Game, but choose a different target number each day, perhaps 200, 500, or 1000. Gradually make the game more challenging by including negative numbers, fractions, and decimals. For example, you could make your target number 1 and find pairs such as .35 and .65, or ¼ and ¾. Continue to discuss strategies with your partner.

Mental Math Set B: Rounding and Estimation Games

For your second week of mental math, you will strengthen your mental rounding and estimation skills through some friendly, fast-paced competition.

Day 1: With a partner, take turns giving each other a number, along with a place value to round to. For example, you might say "780. Round to the nearest hundred." Your partner would answer with "800." Gradually make the problems harder by including decimals and fractions, such as 32⅔ rounded to the nearest tenth. (Answer: 32.7.) Try to think faster as you go, increasing the pace of your game. If you do not have a partner, jot down several numbers and place values to round to, then race the clock to see how many you can answer in three minutes.

Day 2: Either play this game in a store or refer to a grocery store sale flyer. You can play it alone or with a friend. Make a grocery list (real or imagined) with several items. Also list the price of the item. Mentally round and sum the prices to get an estimate of the total cost of the items. (Bonus points if you have coupons that you can mentally deduct from the total and use to save money!) Then compare the estimate with the actual value, which you or your friend can find using a calculator. If playing against a friend, see which of you can come closer to the actual price.

Days 3 and on: Repeat the previous activities, making the problems more and more challenging, and the pace quicker and quicker. Try making calculations involving multiplication and division, too. Can you feel your mental math muscles getting stronger?

Assignments

Textbook Assignments and Test

1. Read textbook sections 1.1 through 1.3 in *Math for Your World* (Blitzer 2016). For each section, follow along with the examples and try the Checkpoint problems. Check your answers against those at the back of the textbook. Verbally answer the Concept and Vocabulary Check exercises at the end of the section. Check your answers with those at the back of the book.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 1.1 through 1.3 (choose from the odd-numbered problems only). Choose several problems of each type to ensure sufficient practice.
3. Do all odd-numbered Application Exercises for Exercise Set 1.2 and 1.3 (no Application Exercises are necessary for Exercise Set 1.1). Check your answers with the back of the book. Make any necessary corrections and review areas that need work. If you need additional practice, you may want to complete a selection of even-numbered problems.
4. Review the Chapter 1 Summary at the end of the chapter. If you feel you need additional practice in any area, select problems from the Chapter 1 Review at the end of the chapter.
5. Complete the Chapter 1 Test from the textbook (for independent students) or the Lesson 1 Test from the test packet (for enrolled students). Students who are using the curriculum independently will complete the test in the textbook and check their answers in the back of the book. Make necessary corrections and review areas that need work. Students who are enrolled in Oak Meadow School must complete the Lesson 1 Test from the test packet and submit it to their teacher for grading.

Reminder for enrolled students: You are encouraged to use the Chapter 1 Test in the textbook as a practice test. This will allow you to check your answers in the back of the book to ensure that you have mastered all major concepts in the chapter. If there are any areas that need work, review and practice as needed before taking the Lesson 1 Test from the test packet.

Math Journal

Complete both journal assignments (do one per week):

- Journal A: Math and You
- Journal B: The Four Step Problem Solving Method in Action

If there is a problem you can't solve, then there is an easier problem you can solve: find it.

George Pólya, *How to Solve It*

Journal A: Math and You

This assignment will help you explore your relationship with mathematics while introducing you to math journaling.

Consider the following questions: When you hear the word "math," what comes to mind? What experiences have you had in learning math? In what ways do you use math outside of math class? If you could study or do anything involving math, what would it be? If you could meet any mathematician, past or present, who would it be?

Choose one or more of these questions and begin to write. Don't worry about the mechanics of your writing; just let your ideas flow for five to ten minutes. When you finish, read your response and quietly reflect on it for a moment. Have you ever really thought about these questions before? Do your answers surprise you?

Journal B: The Four-Step Problem-Solving Method in Action

A great feature of the four-step problem-solving method is that it's useful for solving more than just math problems! In this assignment, you will apply the method to solving a problem in your own life.

For this journal entry, do the following:

1. Give an example of an everyday problem you might encounter.
2. Devise a plan for solving the problem using Pólya's four-step problem-solving process. (Review section 1.3 in the textbook if necessary.)
3. Explain how you might carry out your plan. Be sure to list each of Pólya's four steps and briefly describe how it applies to your problem.

   If you're stuck for an idea for a problem, try some of these ideas: how would you go about organizing a notebook, planning out your schedule, or determining the fastest driving route to a friend's house? These are just a few suggestions; you are encouraged to come up with your own ideas.

Activities

Complete all three activities below.

- Activity A: Modeling College Graduation Rate
- Activity B: Figurate Numbers and Pascal's Triangle
- Activity C: Logic Puzzles

Activity A: Modeling College Graduation Rate

In section 1.2, you explored how to create a linear model for graphed data and use it to make estimates. In this activity, you will create your own bar graph, develop a model that estimates the relationship between the variables, and use your model to make a prediction.

Percentage of College Graduates Among People Ages 25 and Older, in the United States

Year	1940	1950	1960	1970	1980	1990	2000	2010
Percentage	4.6	6.2	7.7	11.0	17.0	21.3	25.6	29.9

Source: U.S. Census Bureau

Make a bar graph for the data in the chart above. Place the years on the horizontal axis and percentage of college graduates on the vertical axis. Be sure to give the graph a title and label the axes. Use a straightedge and graph paper to create neat lines. Tip: Refer to example 8 in section 1.2 for a similar problem.

1. Estimate the increase in the percentage of college graduates per year. Round your answer to the nearest tenth of a percent. Show all of the steps you took to get your answer.
2. Now write a mathematical model (an equation) that estimates y, the percentage of Americans age 25 and older who graduated college x years after 1940.
3. Using your model from step 2, project the percentage of college graduates for the year 2020. Again, show your steps.

Activity B: Figurate Numbers and Pascal's Triangle

Pythagoras of Samos, the man for whom the famed Pythagorean Theorem was named, was a mathematician in Ancient Greece. He began a society called the School of Pythagoras, where students came to learn about math and Pythagoras' mystical beliefs. Among the teachings of the Pythagoreans was the idea that numbers were sacred and some numbers were more special than others. In this activity, we will explore certain "special" numbers called figurate numbers and look for patterns. (We will learn more about the life of Pythagoras in lesson 10.)

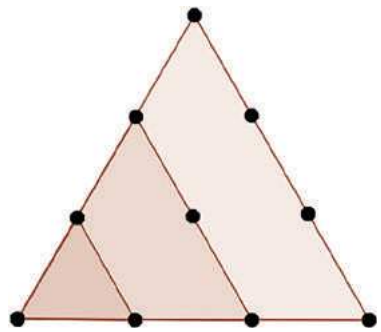
Figurate numbers are special because they can be represented by an arrangement of equally spaced dots in a regular geometrical shape. In other words, they are the number of evenly spaced dots needed to form both the outline and interior of a particular shape. You were introduced to *figurate numbers* in the Application Exercises for section 1.1. Now we will dig a bit deeper into the patterns formed by these special numbers. This activity has two parts.

Part 1

Important note: The number 1 is defined to be the first figurate number for all shapes, as a single point could form the basis for any shape.

1. If the pattern of the number of dots forms a triangle, then the number of dots is called a **triangular number**. The number 3 is a triangular number because three evenly spaced dots form a triangle. Likewise, the numbers 6 and 10 are also triangular numbers.

   Viewing tip: In the figure below, start by looking at the darkest triangle, which contains three dots. Now look at the triangle formed by overlapping the darkest triangle and the slightly lighter region. This second triangle contains six dots. Finally, look at the largest triangle which contains all the points from the first two triangles. Notice that it contains ten dots, nine on its exterior and one on its interior.

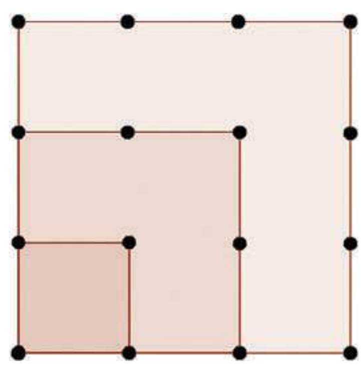


Triangular numbers: 1, 3, 6, 10,...

Look for a pattern in the first four triangular numbers listed in the caption above. (Hint: note a pattern with the difference between each number and the following number.) Describe your pattern and find the next four triangular numbers. You may find it helpful to extend the figure so that larger triangles are formed. This will allow you to count the dots and verify the pattern.

2. A pattern of dots that forms a solid square gives us the square numbers.

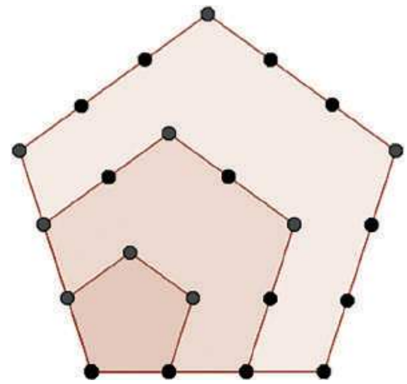
   Viewing tip: Start by looking at the darkest square in the figure below and notice that it contains four dots. Then look at the medium-sized square that overlaps the smallest square. This contains nine dots. Finally, look at the largest square formed by overlapping the smaller squares, and verify that it contains sixteen dots, twelve on its exterior and four on its interior.



Square numbers: 1, 4, 9, 16,...

Look for a pattern in the first four square numbers listed in the caption above. Find the next 4 square numbers and describe your pattern.

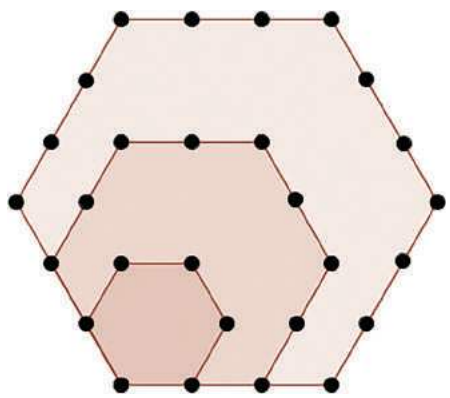
3. Similarly, the pentagonal numbers are generated by the pattern of dots forming regular pentagons.



Pentagonal Numbers: 1, 5, 12, 22, . . .

Look for a pattern in the first four pentagonal numbers listed in the caption above. Find the next 4 pentagonal numbers and describe your pattern.

4. And the pattern of dots forming regular hexagons give us the hexagonal numbers.



Hexagonal Numbers: 1, 6, 15, 28, . . .

Look for a pattern in the first four hexagonal numbers listed in the caption above. Find the next 4 hexagonal numbers and describe your pattern.

Part 2

Once you have done that, you are ready to explore more patterns with the figurate numbers using **Pascal's Triangle**. Pascal's Triangle is a triangular arrangement of numbers that was named for mathematician Blaise Pascal (1623–1662), who studied it extensively. (We'll learn more about Blaise Pascal in lesson 11.) The triangle itself was discovered long before Pascal was even born, though; a version from 13th century China has been found!

The arithmetic triangle, as Pascal's Triangle is also known, is constructed by placing the number one at the top of a page with two ones in the row beneath it. Each successive row is formed with one more entry than the row above it, a 1 at the very left-most entry and the very right-most entry. Each of the remaining entries is the sum of the two numbers directly above it. (See the diagram below.)

Pascal's Triangle has many interesting properties, enough to take up an entire course! For now, what we are interested in is the patterns formed by the figurate numbers in Pascal's Triangle. Your mission is to locate some of them.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

The first 11 rows of Pascal's Triangle

1. The diagram above shows the first 11 rows of Pascal's Triangle. Following the pattern, write down the 12th row.
2. The triangular numbers are the easiest to find in Pascal's Triangle. Using your list of triangular numbers from Part 1, look for them in the triangle. Use a red pen or pencil to circle the triangular numbers on the diagram. (If you need a hint, look at the diagonals.) Describe the pattern in Pascal's Triangle for triangular numbers. Does your pattern verify the next four triangular numbers you identified in Part 1, step 1?
3. The hexagonal numbers are in a similar location in Pascal's Triangle. Using your list of hexagonal numbers from Part 1, look for them in the triangle. Use a blue pen or pencil to circle the hexagonal numbers on the diagram, making sure to also leave previous markings visible. Describe the pattern in Pascal's Triangle for hexagonal numbers. Does your pattern verify the next four hexagonal numbers you identified in Part 1, step 4?
4. The square numbers and pentagonal numbers can also be found on Pascal's Triangle, but they are not quite as obvious because they require adding entries together in the triangle. To see how these work, and to learn about other applications of Pascal's Triangle to patterns of numbers, visit http://www.mathsisfun.com/pascals-triangle.html. (This link, along with all the other links mentioned in this course, can be found in clickable form on the Math Connections resource page

on the Oak Meadow website at www.oakmeadow.com/curriculum-links/. Bookmark that page for easy access to all the recommended online sources!)

Activity C: Logic Puzzles

Choose and complete **one** of the following activities. **Enrolled students:** Earn extra credit by doing both of these activities.

1. **Sudoku:** Read the explanation for solving a Sudoku puzzle at the top left of page 40 in your textbook. Copy the puzzle grid into your notebook and solve the puzzle by filling in the grid as described. Write a one-paragraph reflection on your experience with solving a Sudoku puzzle.

   Note: If you are not familiar with Sudoku puzzles and you want to start off by trying some easier puzzles, go to http://www.sudoku.com and choose "Beginner" from the "Difficulty" menu. A beginner-level puzzle will be generated for you to practice with. Scroll down to the bottom of the page for some great explanations and tips.

2. **Magic Squares:** Read the explanation and example for solving a Magic Square in the middle of the left-hand column on page 40 of the textbook. Complete exercise #48, parts a and b, on the same page. Write a one-paragraph reflection on your experience with solving a Magic Square.

   If you enjoy this type of puzzle, check out the links for Magic Square puzzles in the Additional Resources section at the end of the chapter. You can even learn how to create your own Magic Squares!

Going Further: Additional Resources

If you enjoy Sudoku puzzles, you may also enjoy Kenken and Kakuro puzzles. Check out the links for Sudoku, Kenken, and Kakuro puzzles in the Math Connections resource page on the Oak Meadow website www.oakmeadow.com/curriculum-links/. You'll find other links and resources for this lesson there as well.

FOR ENROLLED STUDENTS

Once you have finished lesson 1, please submit the following items to your Oak Meadow teacher:

- Math journal entries A and B
- Activities A, B, and C
- Lesson 1 Test (from the test packet)

If you completed the Chapter 1 Test in the textbook for practice, do not include that with your submissions. Your teacher only needs to see the test from the test packet. You do not have to submit Practice Exercises or Application exercises either.

Be sure your submission meets all criteria listed in your teacher's welcome letter. Refer to the course checklist for specific guidelines.

People in Mathematics

Image source: Math.info

George Pólya (1887–1985) was a Hungarian mathematician who did not love mathematics in his youth but went on to literally write the book on problem solving. How to Solve It has sold more than a million copies since it was published in 1945 and is still widely used today.

Archimedes of Syracuse (287–212 BCE) is considered one of the greatest mathematicians of all time. Many of his discoveries in geometry, such as the volume and surface area of a sphere, are still used today. He is most famous for solving problems through his inventions, and for a story in which he made a brilliant scientific discovery while in the bath and ran through the streets naked, yelling "Eureka!" To learn more about the legendary Archimedes, check out the three fascinating video links in the Math Connections resources page on the Oak Meadow website.

Lesson

2

Set Theory

When you were very young, you learned how to categorize objects into "sets," or collections, by characteristics such as size, shape, and color. As you have grown, so has your ability to classify abstract concepts using more sophisticated methods. In this lesson, we will explore ways of categorizing and representing sets, including the use of set notation and Venn diagrams. This will help you organize and make sense of information and enable you to more effectively solve problems.

This lesson should take approximately three weeks to complete.

Learning Objectives

- Represent a set using a description, the roster method, and set-builder notation.
- Distinguish between finite and infinite sets.
- Recognize subsets and the empty set, and use appropriate notation.
- Represent set relationships using Venn diagrams.
- Find the complement of a set, and the intersection and union of two sets.
- Perform operations with sets.
- Use Venn diagrams to solve problems.
- Apply knowledge of set operations and Venn diagrams to create and solve problems.
- Identify a mathematician of personal interest.

ASSIGNMENT SUMMARY

- ☐ Mental Math Set A: Grouping Strategies
- ☐ Mental Math Set B: Multiplication Strategies
- ☐ Mental Math Set C: Division Strategies
- ☐ Read Chapter 2 in textbook.
- ☐ Complete a selection of exercises for sections 2.1 through 2.5.
- ☐ Read Chapter 2 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: How Big is Infinity?
- ☐ Math Journal B: Activity Reflection
- ☐ Complete mathematician project proposal.
- ☐ Activity A: Blood Types and Venn Diagrams
- ☐ Activity B: Create Your Own Survey Problem

Why It Matters

> *A set is a Many that allows itself to be thought of as a One.*
>
> **Georg Cantor**

Set theory involves classifying collections of information in order to understand relationships among the collections. This provides order and structure for many areas of mathematics, including algebra and topology, and is also useful for applications in computer science, biology, the social sciences, and business. For example, in algebra, when solving equations, we need to know which set of numbers we are working with—the real numbers, the integers, whole numbers, and so on—before we can find the solution. In computer science, programmers use set theory to set up searches of categorized information in databases. Whenever you enter a search term on the Internet, set theory is at work behind the scenes!

Mental Math Warms-ups

This lesson contains three sets of mental math warm-ups. Complete one set each week.

- Mental Math Set A: Grouping Strategies
- Mental Math Set B: Multiplication Strategies
- Mental Math Set C: Division Strategies

Mental Math Set A: Grouping Strategies

Can you calculate 274 + 198? Sure, you could pull out paper and pencil and add the three-digit numbers, carrying ones along the way. But there is an easier way! We can use the estimation techniques from lesson 1 to make exact calculation simpler.

If the problem had asked you to mentally calculate 274 + 200, it would not have been quite so daunting, would it? We would just add 2 to the hundreds column of 274, getting the answer 474. Notice, though, that 198 is awfully close to 200. In fact, 200 is a great estimate for 198. If we change the problem and add 200 instead of 198, we'll get an estimated answer that is pretty close to the true answer. But how close? Well, 200 is greater than 198 by two and we added it to 274 to get 474. That means our estimate is too high by 2. Therefore, if we subtract 2 from the estimated result of 474, we will get the exact solution 472.

Written mathematically, here is what we did:

$$274 + 198 = 274 + (200 - 2) = (274 + 200) - 2 = 474 - 2 = 472$$

We realized that 198 is equal to 200 – 2, so we substituted 200 – 2 for 198. Then we re-grouped the numbers so we could easily add them.

In this week's mental math exercises, we will use similar grouping "tricks" to make it easier to solve problems mentally.

Day 1: With a partner, take turns giving each other an addition problem involving two-digit numbers. Make sure one of the numbers is close to an easy-to-work-with number. For example, 19 is a good choice because it is close to 20, so you might give the problem 25 + 19. Use the grouping trick to mentally find each sum and briefly explain to your partner how you changed the grouping of the original problem. For example, since 19 is one less than 20, we can mentally add 25 + 20, getting 45, and then subtract 1 since our approximation is too large by 1. Mathematically, it looks like this:

$$25 + 19 = 25 + (20 - 1) = (25 + 20) - 1 = 45 - 1 = 44$$

Gradually make the problems harder by using numbers with three, and perhaps even four, digits. If you do not have a partner, jot down several problems and see how many you can answer in two minutes.

Day 2: Today we will try the same activity with subtraction, but first let's look at whether it works the same way. Let's take, for example, 250 – 98. A good approximation for 98 is 100, so we can subtract 100 from 250 to get 150. We subtracted 100 instead of 98, so our answer is too large by 2. That means our answer is 152. Mathematically, we computed the following:

$$250 - (100 - 2) = (250 - 100) + 2$$

Notice that while we started with 100 – 2, when we regrouped we had to add 2 due to the distributive property over subtraction. Likewise, if we compute 250 – 102, we would subtract 2 because of the distributive property.

$$250 - (100 + 2) = 250 - 100 - 2 = 148$$

Repeat the exercise from Day 1 with subtraction problems. If it helps, use a calculator to quickly verify your answers.

Day 3: Does grouping also work for multiplication? Let's check it out. Suppose we want to find 200 × 21. Rather than doing out two-digit multiplication, we can approximate the answer using 20 instead of 21. 200 × 20 is twice the value of 200 × 10, so we get 2 × 2,000, or 4,000. We know this approximation is lower than the true solution because we multiplied 200 by 20 instead of by 21. If we think of multiplication as groups of items, we calculated the number of items in 20 piles of 200 when we actually needed to find out how many items were in 21 piles of 200. That means we need to add one pile of 200 to our answer, so we get 4,000 + 200 = 4,200. Mathematically, we regrouped and applied the distributive property over addition like this:

$$200 \times 21 = 200 \times (20 + 1) = (200 \times 20) + (200 \times 1) = 4{,}000 + 200 = 4{,}200$$

Repeat the previous days' exercises with multiplication problems, being sure to use "convenient" numbers that make mental calculation simple.

Days 4 and 5: Repeat the previous three exercises, mixing things up with addition, subtraction, and multiplication problems. Challenge yourself or your partner by choosing slightly less convenient numbers.

Mental Math Set B: Multiplication Strategies

Imagine you are planning a party with your friends and you need to buy a dozen drinks that cost $1.50 each. How can you quickly tell how much the drinks will cost? In this week's mental math exercises, you will learn some techniques to help you easily solve this problem and other multiplication problems.

Day 1: To solve the problem with the drinks, a helpful multiplication trick is to double and halve. In other words, when you are finding the product of two factors, you can double one of the factors and halve the other. This is valid because multiplying the expression both by 2 and $\frac{1}{2}$ means we are actually multiplying by $\frac{2}{2}$ which is 1, and not really changing the problem. Choosing which factor is doubled and which is halved can make a difference in terms of how "nicely" the calculation works out. For instance, in our example we want to calculate 12 × $1.50. We could double 12 and halve $1.50, getting the product 24 × $0.75. But is that any easier to calculate than the original problem? Not really. However, if we double $1.50 and halve 12, we get 6 × $3, which is a much simpler problem to solve! We can easily see that our product is $18. With a partner, take turns giving each other multiplication problems to solve. Estimate as needed.

Day 2: Try using a trick to multiply prices that end in 99 or 98 cents. If you are buying 4 notebooks that each cost $2.99, we can calculate the price of 4 notebooks costing $3 to be $12, and then subtract the 4 extra cents, getting the total cost of $11.96. Mathematically, we used the distributive property over subtraction like this:

$$4 \times \$2.99 = 4(\$3.00 - \$0.01) = 4 \times \$3.00 - 4 \times \$0.01 = \$12.00 - \$0.04 = \$11.96$$

Practice this technique by making up problems to solve with your partner. The next time you are out shopping, use this mental math trick to figure out how much you will spend.

Day 3: Use "compatible" numbers to simplify multiplication problems with more than two factors. For example, if we need to multiply 20 × 15 × 5, we can change the order of the factors using the associative property of multiplication to first multiply 20 and 5 to get 100, and then multiply 100 by 15, getting 1,500. Since 20 × 5 equals 100, an easy to work with number, 20 and 5 are "compatible." Compatible numbers don't have to yield a product of 100; any easy to work with product can be used for this trick. With a partner, practice giving each other problems with three factors, some of which are "compatible."

Day 4: If none of your factors is compatible, you may be able to break apart the factors into other factors in order to find compatible numbers. For example, 75 × 12 doesn't give us compatible factors, but if we rewrite 75 as the product of 3 and 25, and 12 as the product of 3 and 4, our problem then becomes 3 × 25 × 3 × 4, and by the associative property of multiplication, we can multiply the compatible numbers 25 and 4, giving us 100 × 3 × 3 = 900. With a partner, pose problems to each other with numbers that can be broken down to form compatible numbers.

Day 5: Practice multiplying numbers using all of the techniques you learned in the previous four days.

Mental Math Set C: Division Strategies

> *Science attempts to find logic and simplicity in nature. Mathematics attempts to establish order and simplicity in human thought.*
>
> **Edward Teller**

Division problems come up all the time in real life. When you grab a bite to eat with friends, how do you share the cost? When you order a pizza, how do you split up the slices so everyone gets an equal amount? Which size bottle of shampoo is a better value? Having some tricks on hand will help make mental division quick and easy.

Day 1: Practice division by 10, 100, 1,000, and other positive powers of ten. Division by a positive power of ten is simple because we can just move the decimal point one position to the left for each zero in the divisor. For example, a $45.00 restaurant bill divided among 10 people would cost $4.50 per person. With a partner, take turns giving each other word problems that require division by a positive power of ten.

Day 2: What if the numbers in a real-life division problem aren't quite as "nice" as the powers of ten? Our estimating tricks from lesson 1 can help! Approximate both the dividend and divisor, and then solve. It is helpful to round up for either the dividend or divisor and then round down for the other in order to reduce the total error from estimating the answer. For example, if we are finding the unit price of a 96 fluid ounce bottle that costs $16.29, we can round 96 up to 100 and round $16.29 down to $16, giving us an estimated quotient of 16 cents per ounce. With a partner, take turns posing problems and using estimated quantities in order to mentally do division.

Day 3: Quick, what's 410 divided by 5? Now that you know about the trick for dividing by 10, you can also easily divide by 5! If we were to divide 410 by 10, we would quickly come up with 41. Since 10 is twice as much as 5, our result of 41 is half of what we would get by dividing 410 by 5. In other words, we divided by twice as much as we should have. To correct for this, we can double our result of 41 to get the answer 82. This "divide by 10, then double" rule will always work for division by 5. Mathematically, we substituted $\frac{10}{2}$ for 5:

$$\frac{410}{5} = \frac{410}{\frac{10}{2}} = \frac{410}{\frac{10}{1} \times \frac{1}{2}} = \frac{410}{10} \times \frac{2}{1} = 41 \times 2 = 82$$

With a partner, take turns giving each other problems with division by 5 and solving them by dividing by 10, then doubling. Bonus question: would it also work to double first, then divide by 10? Test it out and see for yourself!

Day 4: Division by 20 can be done by first dividing by 10, and then dividing the result by 2. For example, in order to divide 640 by 20, we can calculate 640 ÷ 10 = 64, and then 64 ÷ 2 = 32. Using math notation, the process looks like this:

$$\frac{640}{20} = \frac{640 \times 1}{10 \times 2} = \frac{640}{10} \times \frac{1}{2} = 64 \times \frac{1}{2} = 32$$

With a partner, take turns giving each other problems with division by 20 and solving them by dividing by 10, then 2.

Day 5: Expand on the ideas above to practice dividing by 50, 200, 500, etc. How can you easily divide by 50? (Hint: How could the trick for division by 5 be adjusted to work for 50?) Use the estimation techniques from Day 2 if the quotient is not a whole number.

Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.

David Hilbert

Assignments

Textbook Assignments and Test

1. Read textbook sections 2.1 through 2.5 in Math for Your World. For each section, follow along with the examples and try the Checkpoint problems. Check your answers with the back of the book. Verbally answer the Concept and Vocabulary Check exercises at the end of the section. Check your answers with the back of the book.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 2.1 through 2.5 (odd-numbered problems only). Choose several problems of each type to ensure sufficient practice.
3. Do all odd-numbered Application Exercises for each Exercise Set. Check your answers with the back of the book. Make any necessary corrections and review areas that need work. If you need additional practice, you may want to complete a selection of even-numbered problems.
4. Review the Chapter 2 Summary at the end of the chapter. If you feel you need additional practice, select problems from the Chapter 2 Review at the end of the chapter.
5. Complete the Chapter 2 Test from the textbook (for independent students) or the Lesson 2 Test from the test packet (for enrolled students). Students who complete the textbook test are encouraged to check their answers in the back of the book, making necessary corrections and reviewing areas that need work. **Students who are enrolled in Oak Meadow School must complete the test from the test packet.**

Math Journal

Complete both journal assignments. Tip: do one per week.

- Journal A: How Big is Infinity?
- Journal B: Activity Reflection

Journal A: How Big Is Infinity?

In Section 2.1 you learned that a finite set is a set with cardinality 0 or a natural number, which means that the set contains 0 or a natural number of elements. A set that is not finite—in other words, its cardinality is not 0 or a natural number—is called an infinite set. There exist an infinite number of infinite sets, but are all infinite sets equivalent (that is, the same size)?

Find out more about infinite sets by watching one (or both) of the following videos.

Dennis Wildfogel's "How Big Is Infinity?" (TED-Ed):
http://ed.ted.com/lessons/how-big-is-infinity#watch

Numberphile's "Infinity is Bigger Than You Think":
https://www.youtube.com/watch?v=elvOZm0d4H0

In your math journal, write a one-to-two paragraph response describing what you learned about infinite sets and sharing your thoughts on the video(s). Be sure to mention which video you watched.

Journal B: Activity Reflection

Do this journal assignment after you complete Activity B (see the activity section of this lesson).

Think back for a minute about how you created your survey problem for Activity B. Write a one paragraph reflection on your experience. Some questions to consider: Was this activity easier or more challenging than you initially thought? Did you encounter any difficulties as you created your survey problem? If so, what did you do to adjust?

Project Milestone

Mathematician Project Topic Proposal

After completing lesson 6, you will wrap up the first semester by writing a paper for your midterm project. This week, you will begin working on your project by creating a proposal.

For your midterm project, you will write a 3–5 page research-based paper on a mathematician of your choice. But there's a twist: the paper must be written from the point of view of your chosen mathematician, as if he or she were writing an autobiography. This will allow you to "get into the head" of your mathematician, like a character actor, and relive his or her life and mathematical discoveries. While the paper must be grounded in fact and will require citations, you have full creative license to interpret the circumstances surrounding the facts and imagine your mathematician's thoughts and feelings. You may choose to write the paper in the form of a letter, diary entries, a memoir, an interview, or even an obituary "written" by your mathematician. Be creative!

This paper should focus on the mathematical work done by your chosen person. Information such as the early life and education is important, but the spotlight should be on the person's mathematical ideas. You are not expected to fully understand this person's work, but you should be able to explain it in general terms.

Since this is a large project, it is best approached by researching, organizing, and writing the paper in stages over the course of the semester, culminating with the final submission after lesson 6. There will be three "milestone assignments" along the way: the topic proposal, the bibliography, and the outline.

Note for enrolled students: each milestone assignment must be approved by your teacher before you move on to the next phase of the project.

In this lesson, you will accomplish the first milestone: choosing a mathematician to write about. The list below contains some popular suggestions, but you are free to select any mathematician, even one not listed. For inspiration, flip through this coursebook and look at the "People in Mathematics" spotlights found in each lesson. There is also an extensive index of mathematicians on the MacTutor History of Mathematics Archive at http://www-history.mcs.st-and.ac.uk/BiogIndex.html.

You are encouraged to choose a mathematician who is new to you, but please make sure you can find sufficient information to write a 3–5 page paper. If you are up for a challenge, choose a mathematician who is female, or from another culture, or someone who is alive and working today.

For this lesson's milestone, select your mathematician and do some preliminary research. This will let you get to know your mathematician and assess whether enough information is available for you to write a paper about that person. Write a two-paragraph topic proposal that includes:

- The mathematician's name and a sentence or two describing this person's mathematical contributions in your own words.
- An explanation of why you selected this person.
- A citation of the source(s) of your preliminary research information.

Important reminder for all students: Use only reputable sources written by an authority on the subject. The MacTutor History of Mathematics Archive mentioned above is an excellent starting point. Reputable encyclopedias and dictionaries of scientific biography are fine, too. Please note that sites like Wikipedia are never acceptable sources for research information. In most cases, student and teacher webpages are not acceptable, either. Also avoid blog posts (unless written by an authority on the subject) and popular TV station sites like bio.com. If you have a question about whether a source is reputable, ask your parent, tutor, or teacher.

Some suggested mathematicians (remember, you are not limited to people on this list):

Muhammad al-Kwarizmi

Archimedes

George Boole

Georg Cantor

Leonardo DaVinci

Charles Dodgson (aka Lewis Carroll)

René Descartes

Albert Einstein

Paul Erdös

Leonhard Euler

Pierre de Fermat

Leonardo de Pisa (Fibonacci)

Leonardo da Vinci

Galileo Galilee

Carl Friedrich Gauss

Sophie Germain

Stephen Hawking

Omar Khayyám

Ada Lovelace

John Nash

Isaac Newton

Emmy Noether

Blaise Pascal

Plato

Pythagoras

George Pólya

Srinivasa Ramanujan

Alan Turing

Activities

Complete the activities below.

- Activity A: Blood Types and Venn Diagrams
- Activity B: Create Your Own Survey Problem

Activity A: Blood Types and Venn Diagrams

The discovery and classification of different antigens in blood revolutionized the field of medicine. Re-read the information about blood types in the Blitzer Bonus box on page 91 of *Math for Your World* and refer to the Venn diagram in Figure 2.23. Note, in particular, that the Venn diagram classifies the eight different blood types by the presence or absence of each of the three antigens A, B, and Rh in red blood cells. Also note that in order to receive blood in a transfusion, "the recipient must have all or more of the antigens present in the donor's blood." This means that "the set of antigens in a donor's blood must be a subset of the set of antigens in a recipient's blood." (Blitzer, 95)

Let's consider, for example, that Marina, who has type A+ blood, needs a blood transfusion. To have type A+ blood, Marina must have antigens A and Rh, but not B, present in her blood. Because she needs to have all or more of the antigens in the donor's blood, Marina cannot receive blood that contains B antigens. Looking at the Venn diagram, we see that this limits Marina to receiving blood only from donors with O+, O-, A+, or A- blood type.

Use the information provided on page 91 to fill in the chart below and then answer the questions that follow.

Recipient Blood Types	Compatible Donor Blood Types
A+	O+, O–, A+, A–
B+	
AB+	
O+	
A–	
B–	
AB–	
O–	

1. A universal recipient is a person who can receive blood from a donor with any blood type. Based on the Venn diagram and your chart above, which blood type does a universal recipient have?
2. A universal donor is a person who can donate blood to a person with any blood type. Based on the Venn diagram and your chart above, which blood type does a universal donor have?

3. Nikki, who has blood type B–, is donating blood. What blood type(s) must a recipient be to receive her blood?
4. Miguel was in a serious accident and needs a blood transfusion. A quick test at the ER lab indicates that he has O+ blood. What donor blood type(s) can he receive?

Activity B: Create Your Own Survey Problem

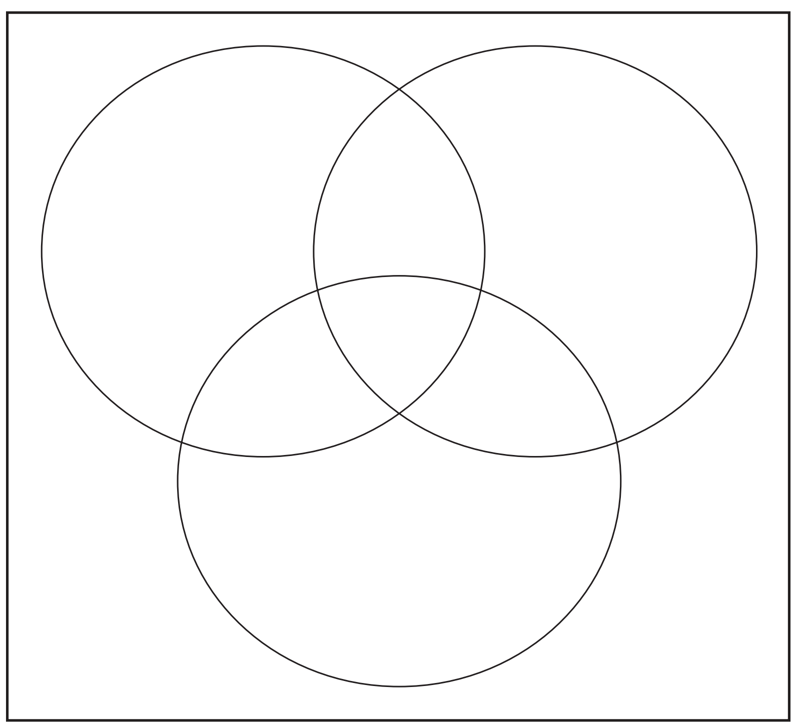
Now it's your turn! Make up your own three-set survey problem about a fictional situation of your choice. Note: this is fictional, so you will not actually conduct a survey.

For the sake of simplicity, assume that you will survey 100 fictional participants. Start by creating a three-set Venn diagram and fill in fictional numbers so that all 100 participants are represented. Be sure to consider including participants in the region outside of the three sets, and don't forget that the intersections of the sets are included in the total number for each set. Remember, the sum of the numbers in all of the individual regions on the Venn diagram should be 100.

The next step is to determine how much information you must present in order to make the problem solvable. What questions will you ask? For inspiration, refer to similar problems in the Application Exercises on pages 103–104 of the textbook, but be sure that you create a unique problem that will require the solver to make a three-set Venn diagram.

Include at least three questions pertaining to your problem. Also provide the solution to your problem, including a completed and labeled Venn diagram illustrating your problem. You may use or copy the empty Venn diagram below.

If possible, briefly explain to a friend or family member how to solve survey problems and have them test out your problem. This will help you determine if you provided enough information to create a Venn diagram and answer your questions.



Going Further: Additional Resources

For additional resources on solving word problems using Venn diagrams, games involving set theory, and more, visit the resource page on the Oak Meadow website.

FOR ENROLLED STUDENTS

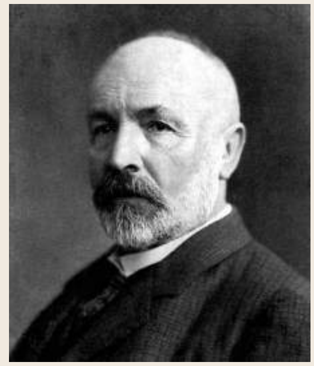
After you submit your topic proposal for your midterm project, please wait for teacher approval before moving on with your research. Once you hear back from your teacher, you should immediately begin searching for sources. This will allow plenty of time for any books you order or request via interlibrary loan to arrive in time to submit your bibliography in lesson 4 (the next project milestone).

Once you have finished lesson 2, please submit the following items to your teacher:

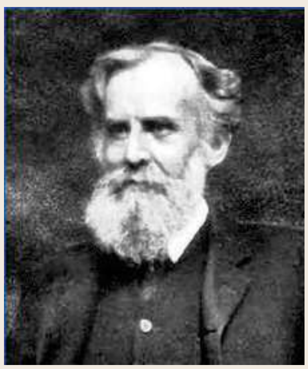
- Journal Entries A and B
- Mathematician Topic Proposal
- Activities A and B
- Lesson 2 Test (from the test packet)

Remember, you do not need to submit any practice or application exercises. Be sure your submission meets all criteria listed in your teacher's welcome letter. Refer to the course checklist for specific guidelines. Please feel free to contact you teacher if you have any questions.

People in Mathematics



Georg Cantor (1845–1918) was a Russian mathematician who is considered the father of set theory. He proved that some infinities are bigger than other infinities. To see this mind-blowing proof in action, check out Vi Hart's video link in the online resources for this lesson.



John Venn (1834–1923) was a British mathematician and logician. He came up with a way to visually represent sets using intersecting discs. Today we refer to his creation as a "Venn diagram."

Emmy Noether (1882–1935) was determined to study mathematics at a time when women in Germany were not permitted to officially do so. She studied anyway and eventually earned her doctorate in mathematics and obtained positions teaching mathematics at several universities. Her work in abstract algebra and theoretical physics earned the respect of other prestigious academics including Einstein. When removed from her job by Nazis because she was Jewish, she again persevered and continued her teaching and research in the United States.

Lesson

3

Number Theory and the Real Number System

Now that we have learned how to classify objects into sets, we can explore how to classify numbers. The number 3, for example, may seem like "just a number." However, it fits into many categories of numbers, including the natural numbers, the whole numbers, the integers, the rational numbers, the real numbers, the prime numbers, and the triangular numbers. In this lesson, we will examine the primary classifications of numbers and we will also explore some unique numerical properties.

This lesson should take approximately three weeks to complete.

Learning Objectives

- Determine whether a natural number is prime or composite.
- Apply divisibility rules to natural numbers.
- Find the greatest common factor and prime factorization for a number.
- Find the least common multiple for two numbers.
- Apply the order of operations.
- Perform operations and solve problems with rational numbers.
- Simplify and perform operations with square roots.
- Classify numbers into sets and subsets.
- Convert between decimal and scientific notation.
- Write terms of arithmetic and geometric sequences.
- Explore recent mathematical discoveries of prime numbers.
- Identify prime numbers and recognize patterns using the Sieve of Eratosthenes.
- Explore some real-world applications of the Fibonacci sequence.

ASSIGNMENT SUMMARY

- ☐ Mental Math Set A: Divisibility Rules
- ☐ Mental Math Set B: Exponents
- ☐ Mental Math Set C: Scientific Notation
- ☐ Read Chapter 3 in textbook.
- ☐ Complete a selection of exercises for sections 3.1 through 3.7.
- ☐ Read Chapter 3 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: Divisibility Trick for Seven
- ☐ Math Journal B: The Largest Known Prime Number (so far)
- ☐ Math Journal C: Fibonacci Connections
- ☐ Activity A: The Sieve of Eratosthenes
- ☐ Activity B: Fibonacci Inspiration

Why It Matters

> *"Numbers are the highest degree of knowledge. It is knowledge itself."*
>
> **Plato**

Studying patterns, classification, and structure of numbers allows us to make sense of them and to apply them in useful ways. For example, breaking numbers down into their prime factorization lets us easily find their least common multiple. This may seem like an academic triviality, but it can have very useful applications. Consider, for example, that two events each happen on a particular schedule, say every 30 days and every 42 days, and you need to determine the next time that they will occur on the same day. Breaking down the numbers using prime factorization and then finding their least common multiple will help you easily solve this problem.

Here is another example: Studying prime numbers may seem like something useful only in math class, but prime numbers are actually at work behind the scenes every time you enter a computer password or make an online purchase. Prime numbers are used by computer programmers to encrypt your data so it remains secure. There really is a practical use for the frenzy behind the quest to find ever-larger prime numbers!

Mental Math Warms-ups

This lesson contains three sets of mental math warm-ups. Complete one set each week.

- Mental Math Set A: Divisibility Rules
- Mental Math Set B: Exponents
- Mental Math Set C: Scientific Notation

Mental Math Set A: Divisibility Rules

Is the number 1,290,354,042 divisible by three? If your first inclination is to grab a pen and paper to start doing long division, wait a minute! There is an easier way to tell if a number is divisible by small numbers.

You are probably already familiar with some divisibility rules. If a number is even, then it is divisible by two. If a number ends in a zero, then it is divisible by 10. And if a number ends in either 5 or 0, then that number is divisible by 5. But have you considered *why* these facts are true?

In order for a number to be divisible by a second number, the original number must contain the second number as a *factor*. (A factor is simply a number that is multiplied by other numbers.) For example, the number 90, which we know is divisible by 10, can be written as 9×10. Since 10 is a factor of 90, 90 is divisible by 10. Likewise, 9 is a factor of 90, so 90 is also divisible by 9. And since we could also write 90 as the product $2 \times 3 \times 3 \times 5$, we can see that 2, 3, and 5 are also factors!

In this week's mental math exercises, we will explore some useful divisibility rules that aren't as commonly known. In your textbook, you'll find a handy reference guide for the divisibility rules (Table 3.1).

Day 1: Practice testing numbers for divisibility by 3. With a partner, take turns giving each other a number and determining whether it is divisible by 3. (Remember, a number is divisible by 3 if the sum of its digits is divisible by 3.) Gradually increase the difficulty by calling out numbers with more digits.

Day 2: Practice testing numbers for divisibility by 4. With a partner, take turns giving each other a number and determining whether it is divisible by 4. (Remember, a number is divisible by 4 if its last two digits are divisible by 4.) Gradually increase the difficulty.

Day 3: Practice testing numbers for divisibility by 6. With a partner, take turns giving each other a number and determining whether it is divisible by 6. (Remember, a number is divisible by 6 if it is divisible by both 2 and 3.) Gradually increase the difficulty.

Day 4: Practice testing numbers for divisibility by 9. With a partner, take turns giving each other a number and determining whether it is divisible by 9. (Remember, a number is divisible by 9 if the sum of its digits is divisible by 9.) Gradually increase the difficulty.

Day 5: Practice testing numbers for divisibility by 12. With a partner, take turns giving each other a number and determining whether it is divisible by 12. (Remember, a number is divisible by 12 if is divisible by both 3 and 4.) Gradually increase the difficulty.

Mental Math Set B: Exponents

Exponents are a regular part of algebra and geometry, so having common squares and cubes memorized will help you work more quickly and accurately.

Day 1: Memorize the perfect squares up to 15^2. With a partner, quiz each other, gradually increasing speed, until you can both quickly recall all of them.

Day 2: Memorize the perfect cubes up to 10^3. With a partner, quiz each other, gradually increasing speed, until you can both quickly recall all of them.

Day 3: Practice estimating the value of square roots by determining two numbers that the square root must fall between. For example, we can estimate that $\sqrt{48}$ is somewhere between 6 and 7 because 48 is between the perfect squares 36 and 49, which have square roots 6 and 7, respectively. With a partner, alternate giving each other square root exercises, and estimating the values.

Days 4 and on: Repeat the exercise for Day 3, but this time try to get even closer to the actual value by estimating how close the number will be to each of the boundary values. For example, 48 is much closer to 49 than to 36, so we can infer that $\sqrt{48}$ will be closer to 7 than to 6. That makes 6.9 a better estimate than, say, 6.2. Challenge your partner to see who can get closer to the actual value of each of your exercises. Use a calculator to check the accuracy of your estimates.

Mental Math Set C: Scientific Notation

Scientific notation is used so often that it is very helpful to be fluent in converting numbers from decimal notation to scientific notation and back again.

Days 1 and 2: Practice mentally converting back and forth from decimal notation to scientific notation by shifting the decimal point accordingly. For example, $3.6 \times 10^4 = 36000$ because the exponent of +4 indicates that we move the decimal point 4 places to the right. Likewise, $6.5 \times 10^{-5} = 0.000065$ because the –5 in the exponents tells us to move the decimal point 5 places to the left. Make up several examples, either alone or with a friend, and mentally convert from one system of notation to the other.

"Perfect numbers like perfect men are very rare."

René Descartes

Days 3 and on: Practice mentally simplifying products and quotients involving scientific notation. Use estimation techniques and the commutative property of multiplication to rearrange the numbers in order to easily simplify problems. For example, a good estimate for $(5.1 \times 10^3)(6.9 \times 10^{-2})$ is $(5 \times 10^3)(7 \times 10^{-2})$. We can then change the order of the factors, like this:

$$(5 \times 7)(103 \times 10^{-2}) = 35 \times 101$$

That gives us an estimated value of about 350. Make up several of your own problems to mentally estimate and solve.

Assignments

Textbook Assignments and Test

1. Read textbook sections 3.1 through 3.7. For each section, follow along with the examples and try the Checkpoint problems. Check your answers with the back of the book. Verbally answer the Concept and Vocabulary Check exercises at the end of the section and check your answers with the back of the book.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 3.1 through 3.7 (odd-numbered problems only). Choose several problems of each type to ensure sufficient practice.
3. Do the following Application Exercises:
   - Exercise Set 3.1: Application Exercises 91 and 95.
   - Exercise Set 3.2: Application Exercises 115, 127, and 129.
   - Exercise Set 3.3: Application Exercises 117, 119, 121, 123, 127, 131, and 133.
   - Exercise Set 3.4: Application Exercises 75, 77, and 79.
   - Exercise Sets 3.5 through 3.7: do all Application Exercises (odds only).

   Check your answers with the back of the book. Make any necessary corrections and review areas that need work. Feel free to complete a selection of even-numbered problems for extra practice.

4. Review the Chapter 3 Summary at the end of the chapter. Select problems from the Chapter 3 Review at the end of the chapter if you feel you need additional practice.
5. Complete the Chapter 3 Test from the textbook (for independent students) or the Lesson 3 Test from the test packet (for enrolled students). Students who complete the textbook test can check their answers in the back of the book, making necessary corrections and reviewing areas that need work. **Students who are enrolled in Oak Meadow School must complete the test from the test packet.**

Math Journal

Complete all three journal assignments (do one per week).

- Math Journal A: Divisibility Trick for Seven
- Math Journal B: The Largest Known Prime Number (so far)
- Math Journal C: Fibonacci Connections

Journal A: Divisibility Trick for Seven

You may have noticed that the table of divisibility rules in your textbook skipped the number 7. Does that mean there is no rule for divisibility by 7? As a matter of fact, a rule for divisibility by 7 does exist, but it's a little too complicated to list in the table.

Watch Dr. James Tanton's video, "Divisibility Rule for 7." The first half of the video demonstrates the rule and the second half proves why it works. Learn the trick and try to follow the algebra in the explanation for why it works.

https://www.youtube.com/watch?v=LPgAK7whEuw

Briefly describe in your own words, the steps needed to test a number for divisibility by 7. Write down any six-digit number and, using the rule, test it for divisibility by 7. Be sure to show your steps.

Journal B: The Largest Known Prime Number (so far)

Many mathematicians throughout history, including Marin Mersenne and Sophie Germain, have devoted their lives to searching for prime numbers through painstaking calculations done by hand. Mathematicians of today continue the search for new prime numbers, but with modern computer technology, the playing field has greatly changed. Since the beginning of the 21st century, prime numbers have made news headlines again and again as the title of "largest prime number" has passed from one unimaginably large number to the next.

While the study of prime numbers may seem frivolous, the fact is that they are essential to our digital world. Prime numbers form the basis for encryption of information passed through the Internet, including credit card and banking information.

For this assignment, research a news story announcing the largest prime number to date and write a brief summary of your findings as if you were explaining it to a friend. Some questions you might consider: What is the largest prime number? When was this number discovered? What tool was used in its discovery? How many digits does the number have? Is there anything else special about this number? Can humans even comprehend a number of that size? Be sure to cite your reference article.

Journal C: Fibonacci Connections

The Fibonacci sequence is a famous sequence of numbers with many applications. Watch Scishow's video, "The Fibonacci Sequence: Nature's Code," at http://youtu.be/wTlw7fNcO-0. Write a 2–3 paragraph journal entry summarizing what you learned about Leonardo de Pisa, the Fibonacci sequence, the Golden Ratio, and how all of this relates to nature.

Activities

Complete both activities below.

- Activity A: The Sieve of Eratosthenes
- Activity B: Fibonacci Inspiration

Activity A: The Sieve of Eratosthenes

You have already learned that a prime number is a natural number greater than 1 that has exactly two natural number factors, namely 1 and itself. You also learned that a composite number is a natural number greater than 1 that has more than two natural number factors.

Suppose we want to find all of the prime numbers between 1 and 100. We could examine each number between 1 and 100, trying to find factors as we did in Section 3.1, but that would be very tedious and time-consuming. Good news: there is a better way!

The Greek mathematician Eratosthenes came up with a method of finding prime numbers, which we call today the Sieve of Eratosthenes. A sieve is a strainer or sifter that lets small items pass through while catching big items, just like a kitchen colander. In this case, the sieve is filtering out (crossing out) composite numbers and keeping (circling) prime numbers. Eratosthenes' method uses a chart of numbers and the instructions below. Follow the instructions, crossing out and circling numbers on the chart as indicated, and then answer the questions.

> *"The moving power of mathematical invention is not reasoning but imagination."*
>
> **Augustus de Morgan**

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 1: Cross out the number 1. (Remember, 1 is neither prime nor composite!)

Step 2: Circle 2 because it is a prime number. Then cross out every multiple of 2 on the chart. (Why? Because any number that is a multiple of 2 has 2 as a factor and is, therefore, composite.) This eliminates all of the even numbers except for 2.

Step 3: Circle 3 because it is the next prime number. Then cross out every multiple of 3 on the chart, as these numbers will also be composite.

Step 4: Circle 5 because it is the next prime number. Then cross out every multiple of 5 on the chart.

Step 5: Continue in this way (circle 7 and cross out its multiples, and so on) until every number from 1 to 100 has either been circled or crossed out. The circled numbers are all the prime numbers from 1 to 100.

After you have done that, answer the following questions.

1. Why did the instructions have you skip checking the number 4 and its multiples?
2. List all of the prime numbers between 1 and 100. How many primes are there between 1 and 100?
3. When exploring mathematics, we always look for patterns. One pattern you may have noticed is that for each number whose multiples you crossed out, you did not have to cross out any multiples until you reached the square of that number. For example, when you crossed out multiples of 3, you circled 3 because it was prime, but 6 was already crossed out. 32 = 9 was the first multiple that needed to be crossed out. Likewise, when you crossed out multiples of 5, the multiples 10, 15, and 20 were already crossed out, leaving 52 = 25 as the first multiple to be crossed out. Why is this the case? (Hint: think about what numbers are divisors of the numbers already crossed out.)
4. What was the largest prime number from 1 to 100 that had a multiple that still needed to be crossed out? In other words, after which prime number could you stop checking for multiples? How did you know for certain that there would be no more primes between 1 and 100? Try to base your explanation on the pattern mentioned in question 3.

5. Suppose you were asked to use the Sieve of Eratosthenes to find all prime numbers between 1 and 400. What would be the largest prime number you would need to check before you could stop crossing out composite numbers and declare all prime numbers found?
6. Apply what you have learned through this activity to find the next five prime numbers greater than 100.
7. An *emirp* (prime written backward) is a prime number whose digits can be reversed and form a different prime number. The number 13 is the smallest emirp. The reverse of its digits is 31, which is also a prime number. What other emirps do you see on your chart of prime numbers?
8. If a prime number is doubled and increased by one, and the result is a prime number, then the result 2p+1 is called a Sophie Germain prime. For example, 11 is a Sophie Germain prime because it is the result when the prime number 5 is doubled and increased by 1, but the prime number 19 is not a Sophie Germain prime because 19=2(9)+1 and 9 is not prime. Select any three prime numbers under 100 and test to see if they are Sophie Germain primes. Show your steps.

Activity B: Fibonacci Inspiration

Now that you have learned about the Fibonacci sequence, watch Cristóbal Vila's short video, "Nature by Numbers," which visually explores some of the connections of the Fibonacci sequence to nature.

https://www.youtube.com/watch?v=kkGeOWYOFoA

Afterward, reflect on which aspect of the video you found most interesting and, using that as inspiration, create a poem, short story, drawing, painting, choreographed dance, original musical composition, or any other creative work. Include a brief written explanation of how your creative piece features the connection between nature and the Fibonacci sequence. For additional inspiration, check out the links on the Math Connections resources page on the Oak Meadow website.

Going Further: Additional Resources

For additional resources on the Fibonacci sequence, the Sieve of Eratosthenes, scientific notation, exponents, and more, visit the resource page on the Oak Meadow website.

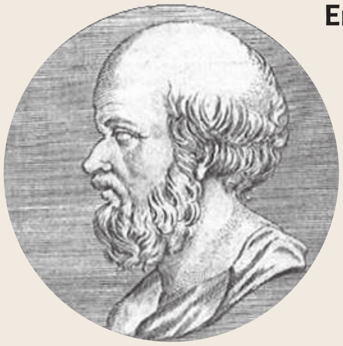
FOR ENROLLED STUDENTS

Once you have finished lesson 3, please submit the following items to your teacher:

- Journal entries A, B, and C
- Activities A and B (let your teacher know if you completed the optional part of Activity B for extra credit)
- Lesson 3 Test (from the test packet)

Make sure your submission is complete before sending it to your teacher.

People in Mathematics



Eratosthenes (276–194 BCE), who was born in Northern Africa, was a scholar and librarian at the Library of Alexandria in Egypt. Among his many mathematical and geographical contributions, he calculated with impressive accuracy the circumference of the Earth by comparing the lengths of shadows in two different cities. He also worked extensively with prime numbers and his Sieve of Eratosthenes is still used today in number theory.

Sophie Germain (1776–1831) was so determined to become a mathematician despite opposition from her parents that as a teenager, she taught herself to read Latin and Greek and endured cold nights huddled under blankets with her math books. Her family of wealthy French merchants, who did not approve of women studying, had confiscated her light, fire, and clothing in an effort to make her stop learning. Young Sophie persevered and submitted a paper under a pseudonym to the well-respected mathematician Lagrange. He was so impressed that he searched for the paper's author, and when he discovered Sophie had written it, he decided to sponsor her. She spent much of her career working on prime numbers and Fermat's Last Theorem.

Paul Erdös (1913–1996) was a child prodigy born to a Jewish Hungarian family. He grew up in the midst of World War I and fled to the United States in World War II. He loved numbers and mathematics to the point that he could not be bothered with anything else, even taking care of himself. He spent most of his life traveling around, living as a guest of other mathematicians and collaborating on many problems in number theory and combinatorics. In fact, he wrote or contributed to an incredible 1,475 math papers, and his fans developed a system of "Erdös numbers" to describe a person's academic relationship to him for bragging purposes. A person with Erdös number 1 published a paper with Erdös, while a person with Erdös number 2 published a paper with a person who published a paper with Erdös, and so on.

Lesson

4

Algebra: Equations and Inequalities

In this lesson, we will revisit some of the algebraic concepts you encountered in Algebra I. We will review how to evaluate algebraic expressions and solve linear equations and inequalities in one variable, as well as how to set up proportions. We will then apply these techniques to creating algebraic models in order to solve application problems. In the activities, we will also ponder the nature of mathematics and try to solve a famous math problem from history.

This lesson should take approximately three weeks to complete.

Learning Objectives

- Evaluate and simplify algebraic equations.
- Use mathematical models to solve problems.
- Solve a formula for a specified variable.
- Compute percent discounts and increases.
- Solve problems using proportions.
- Solve problems involving direct or indirect variation.
- Solve problems involving linear inequalities.
- Locate reputable sources on a chosen mathematician.
- Evaluate the validity of a given solution to a problem.
- Design original problems to be solved using linear equations and inequalities.
- Consider the nature of mathematics.

ASSIGNMENT SUMMARY

- ☐ Mental Math Set A: Percentages
- ☐ Mental Math Set B: Algebraic Translation
- ☐ Mental Math Set C: Beat the Calculator
- ☐ Read Chapter 4 in textbook.
- ☐ Complete a selection of exercises for sections 4.1 through 4.6.
- ☐ Read Chapter 4 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: Percentage Problem OR Math Journal B: Is This Correct?
- ☐ Complete mathematician project bibliography.
- ☐ Activity A: Create Your Own Word Problems
- ☐ Activity B: Is Mathematics Invented or Discovered?
- ☐ Activity C (optional): Diophantus' Riddle

Why It Matters

> *"The only way to learn mathematics is to do mathematics."*
>
> **Paul Halmos**

By using variables and algebraic notation, we can solve problems that would otherwise be cumbersome or confusing to figure out. "Translating" the information in a problem from words into symbols not only helps us make sense of the problem but also allows us to present the ideas in a way that can be followed by anyone familiar with algebra, regardless of language, as long as they use the same number system. The use of algebraic models for real-life situations gives us a powerful way to see what has occurred in the past, what is happening now, and what may happen in the future.

Mental Math Warms-Ups

This lesson contains three sets of mental math warm-ups. Complete one set each week.

- Mental Math Set A: Percentages
- Mental Math Set B: Algebraic Translation
- Mental Math Set C: Beat the Calculator

Mental Math Set A: Percentages

It seems like nearly every store nowadays is running some sort of discount promotion.

Half-price clearance sale!

Everything 30% off!

Save an extra 25% with coupon!

Shopping is a great time to use your mental math skills. In this activity, we will be working to strengthen your skill with mental calculations of percentages.

There are many methods you can use to calculate discounts and sale prices. Any way that works consistently is fine to use. One straightforward method is to calculate the discount—that is, the amount being deducted from the original price—and then subtract that amount from the original price in order to get the sale price (the lower price you would actually pay). For example, a 10% discount on \$100 is \$10, so the sale price is \$100 – \$10 = \$90.

Another way to find discounts and sale prices is to think in terms of the percentage of the original price that you will pay. For example, if an item is 40% off, that's the same as saying you will pay 100% – 40% = 60% of the original amount. The discount is 40%, so the sale price is 60% of the original price. You can then simply multiply the original price by .60 to find the sale price.

After completing this week's mental math exercises, you will have strategies for mentally calculating your own discounts and sale prices right at the store.

Day 1: Work with a partner and create problems for each other, switching between asking for the discount amount and the sale price. Choose "friendly" percent discounts and original prices for this first day, such as 10% and 50% off a $10 or $100 purchase. (Calculate 10% by moving the decimal point one unit to the left; 50% can be calculated by dividing the original price by 2.) Verify your answers with a calculator.

Day 2: Repeat the activity for Day 1, but step it up a notch by using less "friendly" percentages and original prices. Try 25% (which is half of 50%), and multiples of 10%, such as 20% (which is double 10%). Verify your answers with a calculator.

Day 3: Repeat the activity for Days 1 and 2, but challenge yourself even more by using a 75% discount (which is 25% less than the original amount), a 5% discount (which is half of 10%), 15% (which is very useful for calculating tips!), and other multiples of 5%. Can you think of some easy ways to calculate the discounts and sale prices? Verify your answers with a calculator.

Day 4: Repeat the previous activities, but create problems where you know the sale price and original price and need to find the percent discount. Choose "friendly" numbers so you can mentally calculate the percentages. You may want to write down the problem to set it up properly and devise a strategy, but try to make the calculations mentally. Verify your answers with a calculator.

Day 5: Continue with these exercises, but now create problems where you know the sale price and discount amount, but need to find the original price. Again, you may want to write down the problem to set it up properly and devise a strategy, but try to make the calculations mentally. Verify your answers with a calculator.

Mental Math Set B: Algebraic Translation

When solving a word problem, it is necessary to first translate the problem from English into algebraic symbols. The more you practice translating phrases into symbols, the more naturally you will be able to solve word problems. Some frequently used phrases are listed in Table 4.2 in your textbook. Refer to this table for this week's mental math exercises.

Day 1: With a partner, take turns giving each other short verbal word problems that involve addition. Use some of the English phrases listed in Table 4.2 and create your own phrases, if desired. After a player posts the problem, the other player should give an algebraic expression that represents the phrase.

Day 2: Repeat the activity for Day 1 using word problems that involve subtraction.

Day 3: Repeat the activity using word problems that involve multiplication.

Day 4: Repeat the activity using word problems that involve division.

Day 5: Repeat the activity using word problems that involve more than one operation.

Mental Math Set C: Beat the Calculator

For a change of pace, this week you will get to choose the mental math activities you complete. Check out the "BEATCALC" (Beat the Calculator) page on the Math Forum site, which contains a long list of different mental math tricks.

http://mathforum.org/k12/mathtips/beatcalc.html

Choose a new trick to learn each day this week. Amaze your friends and family with your tricks!

Assignments

Textbook Assignments and Test

> *"Algebra is generous; she often gives more than is asked of her."*
>
> **Jean D'Alembert**

1. Read textbook sections 4.1 through 4.6.For each section, follow along with the examples and try the Checkpoint problems. Check your answers with the back of the book. Verbally answer the Concept and Vocabulary Check exercises and check your answers.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 4.1 through 4.6 (odd-numbered problems only). Choose several problems of each type to ensure sufficient practice.
3. Do the following Application Exercises:
   - Exercise Set 4.6: Application Exercises 95–101 (odd-numbered problems only).
   - All other Exercise Sets: do all Application Exercises (odds only).

   Check your answers and make any necessary corrections. Review areas that need work.
4. Review the Chapter 4 Summary at the end of the chapter. If you feel you need additional practice, select problems from the Chapter 4 Review at the end of the chapter.
5. Complete the Chapter 4 Test from the textbook (for independent students) or the Lesson 4 Test from the test packet (for enrolled students). Students who complete the textbook test are encouraged to check their answers in the back of the book, making necessary corrections and reviewing areas that need work. **Students who are enrolled in Oak Meadow School must complete the test from the test packet.**

Math Journal

Choose one of the following math journal assignments (either Journal Assignment A or Journal Assignment B). Be sure to indicate which assignment you chose.

- Math Journal A: Percentage Problem
- Math Journal B: Is This Correct?

Journal A: Percentage Problem

In January of last year, Joe resolved to eat well and exercise, and he lost 10% of his body weight. This year he did not keep up with his healthy habits and gained 10% of his body weight. Does Joe now weigh the same amount he did last January? Why or why not? Give an example weight for Joe and demonstrate your calculations. Be sure to label Joe's weight at the start and end of each year.

Journal B: Is It Correct?

Suppose your friend, Brandon, asks you to check over his math homework. You read his solution to the first problem:

$$-2(x-4) > x+7$$

$$-2x-8 > x+7$$

$$-3x-8 > 7$$

$$-3x > 15$$

$$x > -5$$

Is Brandon's solution correct? If not, point out any mistakes and explain to Brandon where he went wrong and what he should do instead to correctly solve the inequality.

Project Milestone

Mathematician Project Bibliography

As the next milestone for your midterm project, locate at least three reputable sources for your chosen mathematician and create a preliminary bibliography. At least two of your sources must consist of books, journal articles, encyclopedias, etc. (Digital copies of books and articles published originally in print form are acceptable.) Additional sources may include materials found solely online.

Refer to the instructions in the appendix in this course book to learn more about reputable sources and how to create bibliography entries in MLA format. You should also refer to the midterm project instructions in lesson 2 for general information on the project's requirements and a few special notes on acceptable sources.

Activities

Complete activities A and B below. Activity C is optional. (Enrolled students will earn extra credit if they complete Activity C).

- Activity A: Create Your Own Word Problems
- Activity B: Is Mathematics Invented or Discovered?
- Activity C (optional): Diophantus' Riddle

Activity A: Create Your Own Word Problems

As noted in your textbook, "One of the best ways to learn how to solve a word problem in algebra is to design word problems of your own" (Blitzer 2016). In order to create an original word problem, you must understand how much information the reader will need and figure out how to best present that information. By designing your own word problems, you will improve your own skills for solving word problems.

For this activity, create two word problems, one that can be solved using a linear equation and one that can be solved using a linear inequality. Provide both the problem and a fully worked solution.

Activity B: Is Mathematics Invented or Discovered?

Watch the following TED-Ed talk given by Jeff Dekofsky:

> http://ed.ted.com/lessons/is-math-discovered-or-invented-jeff-dekofsky#digdeeper

Take a few minutes to consider your position on this debate. Do you believe that mathematics is a creation of humankind to explain what we observe about the world? Or does mathematics exist as the natural structure of the universe, waiting for people to decipher its secrets? Explain your position in a brief essay of at least three paragraphs. There is no right or wrong answer, but be sure to clearly state your position and provide supporting reasoning in your own words.

If you find this question fascinating or if you are unsure of your position in the debate, you might also want to watch the "Great Math Mystery" NOVA episode, which delves far beneath the surface of this debate:

> http://www.pbs.org/wgbh/nova/physics/great-math-mystery.html

Activity C (optional): Diophantus' Riddle

The Ancient Greek mathematician Diophantus was the subject of a 5th century algebra problem written in the form of an epitaph:

> Here lies Diophantus, the wonder behold.
>
> Through art algebraic, the stone tells how old:

> *"The first rule of discovery is to have brains and good luck. The second rule of discovery is to sit tight and wait till you get a bright idea."*
>
> **George Pólya**

God gave him his boyhood one-sixth of his life,

One twelfth more as youth while whiskers grew rife;

And then yet one-seventh ere marriage begun;

In five years there came a bouncing new son.

Alas, the dear child of master and sage

After attaining half the measure of his father's life chill fate took him.

After consoling his fate by the science of numbers for four years, he ended his life.

How old was Diophantus at the time of his death? Begin by defining your variable and writing an algebraic expression to represent the situation. Solve the equation, showing all steps.

Going Further: Additional Resources

For further resources to help you explore whether you believe mathematics was invented or discovered, as well as information on linear equations and inequalities, proportions, and more, check out the course resource page on the Oak Meadow website.

FOR ENROLLED STUDENTS

After you submit your bibliography for the mathematician project, please wait for teacher approval before moving on to the outline, which will be submitted in the next lesson.

Once you have finished your work for lesson 4, please submit the following to your teacher:

- Journal Entry A or B
- Mathematician Project Bibliography
- Activities A and B
- Optional: Activity C (note; to receive credit, you must give a full algebraic solution to this problem)
- Lesson 4 Test (from the test packet)

Remember, you can contact your teacher at any time if you have questions.

People in Mathematics

Muhammad al-Khwarizmi (circa 780–850) was a Muslim mathematician and astronomer who wrote several books that later introduced Europe to Islamic mathematical discoveries. In one of his books, he explained the Hindu-Arabic numeral system, which is the foundation for the base-ten numeral system we use today. He also introduced the number zero as a placeholder when making calculations. His most famous book was the first algebra book ever written, and the words "al-jabr" in part of its title, gave us the word "algebra."

Diophantus (circa 200–284) was a Greek mathematician who is often called "the father of algebra" in celebration of his contributions to the development of algebra. He wrote *Arithmetica*, a series of thirteen books in which he solved linear and quadratic equations. He is remembered even today for a problem called "Diophantus' Riddle," which you can try to solve in this lesson!

Omar Khayyám (1048–1131) was a Persian poet, mathematician and astronomer. He measured the length of the year with astounding accuracy. He also wrote an algebra book in which he explored cubic equations and geometric connections to algebra.

Lesson

5

Algebra: Graphs, Functions, Linear Functions, and Linear Systems

We will continue our review of topics that should look familiar from Algebra I: graphing points and lines, using function notation, finding equations of lines, and solving systems of linear equations. We will also cover some statistical concepts that you may not have seen before, including correlation and regression lines. Throughout the lesson we will examine various real world applications of these concepts and apply what we have learned to solve problems.

This lesson should take approximately two weeks to complete.

Learning Objectives

- Graph equations and functions on the coordinate plane.
- Identify characteristics of a function based on its graph.
- Calculate and interpret slope.
- Model data using linear equations.
- Create a scatterplot and interpret information from it.
- Solve linear equations by graphing, substitution, and addition.
- Solve problems using systems of linear equations.
- Graph a linear equation in two variables.
- Graph a system of linear inequalities.
- Use mathematical models involving linear inequalities.
- Organize project research notes into an outline.
- Solve application problems.

ASSIGNMENT SUMMARY

- ☐ Mental Math Set A: Visualizing Graphs of Linear Equations
- ☐ Mental Math Set B: Correlation Coefficients and Slopes of Lines
- ☐ Read Chapter 5 in textbook.
- ☐ Complete a selection of exercises for sections 5.1 through 5.5.
- ☐ Read Chapter 5 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: Correlation
- ☐ Complete mathematician project outline.
- ☐ Activity A: Celsius and Fahrenheit
- ☐ Activity B: Literacy and Hunger Statistics

Why It Matters

Graphing offers visual benefits to aid in understanding and solving equations and functions. Using graphing for modeling allows us to see patterns and make predictions. Knowing how to solve systems of linear equations and inequalities gives us very useful tools for solving real-world problems for which there is more than one unknown. Systems of equations can be particularly useful in business situations for determining how much needs to be sold to break even and to generate a profit.

> *"Each problem that I solved became a rule which served afterwards to solve other problems."*
>
> **René Descartes**

Mental Math Warms-ups

This lesson contains two sets of mental math warm-ups. Complete one set each week.

- Mental Math Set A: Visualizing Graphs of Linear Equations
- Mental Math Set B: Correlation Coefficients and Slopes of Lines

Mental Math Set A: Visualizing Graphs of Linear Equations

While understanding the mechanics of how to graph equations is certainly important, it is also important to be able to describe what the graph of an equation will look like without actually graphing it. In this week's mental math exercises, we will exercise our graph visualization skills.

Day 1: With a partner, write a list of several linear equations in slope-intercept form ($y = mx + b$). Now take turns examining each of the equations. What is the value of the slope (m)? If the slope is positive, then the line will rise in an upward direction to the right side of the graph. If the slope is negative, then the line will fall in a downward direction to the right side of the graph. If the absolute value of the slope (ignoring the sign) is greater than 1, then the line is steep; the greater the number, the steeper it is. If the absolute value of the slope is less than 1, then the line is not very steep. If the slope is 0, then the line is horizontal. Now look at the y-intercept (the b in the form $y = mx + b$). This number (including the sign) indicates where the line will cross the y-axis. Based on the information you just gathered, you can give a brief description of whether the line is horizontal, increasing, or decreasing, how steep it is, and where it will cross the y-axis. Continue examining the equations and describing what the graph of the equation would look like.

Day 2: Now take turns with your partner giving each other a description of a graph. The other person should come up with an equation that would fit the description. (Note: there may be more than one correct answer!) For example, if your partner tells you that she is thinking of a graph that is very steep in the positive direction and it crosses through the origin, you could correctly give the equations $y = 5x$, or $y = 10x$, or $y = 402x$. Any answer that correctly fits the description earns a point. Quiz each other to see who can earn the most points for correct answers.

Days 3 and on: Continue repeating the exercises for Days 1 and 2, but include equations in different forms. Try to mentally figure out what the slope and y-intercept will be even when the equation is arranged like $3x + y = 2$. (Remember, the slope will not be 3 because this equation is not in slope-intercept form!) Also include vertical lines in the form $x = a$. Create a bigger challenge for your partner each day.

Mental Math Set B: Correlation Coefficients and Slopes of Lines

Days 1 and 2: Write down several decimal numbers from –1 to 1. Pretend that each of these numbers is the correlation coefficient, *r*, of a data set. Sketch a rough scatterplot for each value, illustrating a possible scenario with that particular correlation. (This is a hypothetical case. Of course we can't know what the scatterplot looks like unless we have the actual data.) For example, a scatterplot with a correlation coefficient very close to zero would show data points in a widely scattered cloud with no discernable direction. Refer to the example graphs in your textbook, if needed.

Day 3 and on: Try to mentally estimate the slope of a best fit line. Choose two points that fall on or very near the best fit line. Approximate the *x* and *y* coordinates for both points, rounding to convenient numbers. Mentally estimate the difference between the *x* values for both points and the *y* values for both points. Divide the difference in *y* values by the difference in *x* values, using mental math techniques, to estimate the slope. Tip: You may want to look through the textbook to find some graphs with best fit lines to use as examples.

Assignments

Textbook Assignments and Test

1. Read textbook sections 5.1 through 5.5. For each section, follow along with the examples and try the Checkpoint problems, checking your answers. Verbally answer the Concept and Vocabulary Check exercises and check your answers.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 5.1 through 5.5 (odd-numbered problems only). Choose several problems of each type to ensure sufficient practice.
3. Do the following Application Exercises:
    - Exercise Set 5.4: Application Exercises 51–61 (odds only).
    - All other Exercise Sets: do all Application Exercises (odds only).

    Check your answers and make any necessary corrections. Review areas that need work.
4. Review the Chapter 5 Summary at the end of the chapter. Use the Chapter 5 Review for additional practice, if necessary.
5. Complete the Chapter 5 Test from the textbook (for independent students) or the Lesson 5 Test from the test packet (for enrolled students). Students who complete the textbook test are encouraged to check their answers in the back of the book, making necessary corrections and

reviewing areas that need work. **Students who are enrolled in Oak Meadow School must complete the test from the test packet.**

Math Journal

> *"All truths are easy to understand once they are discovered; the point is to discover them."*
>
> **Galileo Galilee**

Complete the following journal assignment.

Journal A: Correlation

In your journal, describe a written example of two variables with a strong correlation (positive or negative) where neither variable is the cause of the other. Explain why this is so.

Project Milestone

Organize your research notes in the form of a 1–2 page outline. A typical outline will likely include such headings as:

I. Introduction

II. Early Life

III. Work (There may be more than one section for different periods or types of work)

IV. Later life

V. Conclusion

If you prefer to use a different format or another form of outlining, such as concept mapping, feel free to do so.

Once you have your major headings, fill in topics and details that you want to cover. The outline need not include every detail you will mention, but it should include all major topics and subtopics from the introduction through the conclusion. The more details you fill in now, the easier it will be to write your rough draft.

Please refer to the midterm project instructions in Lesson 2 for the project's general requirements.

Activities

Complete both of the activities below.

- Activity A: Celsius and Fahrenheit
- Activity B: Literacy and Hunger Statistics

Activity A: Celsius and Fahrenheit

Complete Critical Thinking Exercise #68 on page 309 of the textbook. Show all of your steps.

Activity B: Literacy and Hunger Statistics

> *"The essence of mathematics is not to make simple things complicated, but to make complicated things simple."*
>
> **Stan Gudder**

Complete Technology Exercise #60 on page 320 of the textbook. If you plan to use your calculator, simply follow the instructions in the book.

If you do not have a graphing calculator with statistical features, or if you are not comfortable using your calculator's statistical features, use the Line of Best Fit tool on the NCTM Illuminations website:

http://illuminations.nctm.org/Activity.aspx?id=4186.

If you use the Illuminations statistical tool, here are additional instructions to use while completing the textbook exercise (a–e):

- Input your data points as (*x*, *y*) coordinates in the boxes at the bottom left of the tool. Click "Add Point" after entering each point.
- Use the zoom buttons to locate your points in the viewable window of the graph.
- When you reach part c in the textbook instructions, check the "Show line of best fit" box. This will show you the correlation coefficient, *r*, and equation of best fit for your data. This information will allow you to answer parts c and d.
- If you accidentally add an extra point by clicking on the graph, use the X tool to remove it.

Going Further: Additional Resources

For additional resources on graphing, scatterplots, correlations, and other concepts covered in this lesson, visit the resource page on the Oak Meadow website.

FOR ENROLLED STUDENTS

After you submit your outline for the mathematician project, please wait for teacher approval before moving on to writing your paper, which will be submitted after lesson 6. You are encouraged to get started on your paper as soon as you receive approval.

Once you have finished lesson 5, please submit the following items to your teacher:

- Journal entry A
- Mathematician Project Outline
- Activities A and B
- Lesson 5 Test (from the test packet)

Be sure your submission is complete before sending it to your Oak Meadow teacher.

People in Mathematics

René Descartes (1596–1650) was a French philosopher and mathematician. He is remembered for his proposition, "I think, therefore I am," and is often credited with the creation of the Cartesian coordinate system that we still use today to graph points, equations, inequalities, and functions. Descartes is the one who first used the letters *x* and *y* to represent unknown variables.

Galileo Galilei (1564–1642), of Pisa (now in Italy), resisted his father's plans for him to become a doctor and instead studied mathematics. Galileo's observations and experiments led to his discoveries about density, gravity, projectile motion, pendulum motion, musical theory, and astronomy. He created numerous devices, including a telescope. Galileo also realized that the Earth moved around the sun, contrary to the Catholic Church's doctrine that the Earth was positioned at the center of the universe. His persistence in publicizing his discovery angered leaders of the Church. He was found guilty of heresy and sentenced to house arrest for the rest of his life.

Brahmagupta (598–670) was an Indian mathematician and astronomer who studied the planets, eclipses, and the phases of the moon. He also explored the idea of zero, created some algebraic notation, solved certain quadratic equations, and developed a way to calculate square roots.