Math Connections

Teacher Manual

Oak Meadow

Oak Meadow, Inc.
Post Office Box 1346
Brattleboro, Vermont 05302-1346
oakmeadow.com

Introduction

This course is designed to give students opportunities for real-world applications of mathematics. It is intended for students who have already taken Algebra I and Geometry courses. Students will engage in mental math, math journaling, and a wide variety of activities, as well as prepare two large projects.

The textbook, *Math For Your World* (Pearson 2016), is an excellent resource and includes clear explanations of the many concepts and skills covered this year. The answer key at the back of the book includes answers to all odd-numbered practice and application exercises. For this reason, students are assigned these exercises so that they can check their answers and, if mistakes are made, go back and figure out why. In this way, students can direct their own learning and get immediate feedback on their work. Students are also encouraged to choose (with the help of an adult, if necessary) how many practice problems are needed in order for them to become comfortable working with a particular skill or concept.

Students are always required to show their steps when completing a problem. Simply writing the answer is not enough (especially since the answers can be found in the back of the textbook). Showing each step helps the student make sure to proceed logically through a problem, and makes it easy to go back and detect where an error occurred, if necessary.

In this teacher manual, you will find the text for all assignments and some activities, and answers for those activities that pose a specific question. Teacher manual answers are seen in color. If more information is needed about any assignment, you can refer to the textbook or additional instructions in the student's coursebook. You will also find links to many excellent online resources on the Math Connections resource page on the Oak Meadow website at www.oakmeadow.com/curriculum-links/.

If you are homeschooling independently, this teacher manual can serve as your support as you guide and evaluate your student's work. When a student gets a factual answer wrong, you can share the correct answer and address any underlying misconceptions. The focus should always be on the learning process rather than on a sense of judgement. Several incorrect answers related to a particular topic point to an area the student will benefit from revisiting.

For obvious reasons, it is best not to share this teacher manual with your student. Each student is expected to produce original work, and any incidence of plagiarism should be taken very seriously. If you notice a student's answers matching those of the teacher manual word for word, a discussion about plagiarism and the importance of doing original work is necessary. While students in high school are expected to be well aware of academic honesty, any discussion about it should be

approached as a learning opportunity. Make sure your student is familiar with when and how to properly attribute sources.

We encourage you and your student to explore the topics introduced this year through dynamic exchanges of ideas, partnering on mental math activities, viewing and discussing the many video resources mentioned in this course, and in other active, experiential ways. We hope this course leads your student into a better understanding of mathematics and its importance in the wider world.

Lesson

1

Problem Solving and Critical Thinking

Learning Objectives

- Distinguish between and use inductive and deductive reasoning.
- Use estimation techniques to find approximate answers to problems.
- Explain the purpose and features of circle graphs, bar graphs, and line graphs.
- Apply estimation techniques to information presented on graphs.
- Estimate relationships between variables through mathematical modeling.
- Solve problems using the four-step problem solving process.
- Explore puzzles and games that involve problem-solving.

Mental Math Warms-Ups

Each lesson will contain at least one set of mental math activities. These exercises are meant to strengthen your mental math skills over time and to help you learn and discover new techniques to use in your everyday life. They are intended to be quick, so spend just three to five minutes practicing at the beginning of each math session.

You are encouraged to do your mental math exercises together with a partner to enhance learning and make the process more fun. Ask a parent, tutor, sibling or friend to be your partner. You can make it a collaborative effort where you make up and solve problems together with your partner, or you could create some friendly competition by challenging your partner to beat the clock or see who

ASSIGNMENT SUMMARY

- ☐ Mental Math Set A: The Sums Game
- ☐ Mental Math Set B: Rounding and Estimation Games
- ☐ Read Chapter 1 in textbook.
- ☐ Complete a selection of exercises for sections 1.1 through 1.3.
- ☐ Read Chapter 1 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: Math and You
- ☐ Math Journal B: The Four Step Problem Solving Method in Action
- ☐ Activity A: Modeling College Graduation Rate
- ☐ Activity B: Figurate Numbers and Pascal's Triangle
- ☐ Activity C: Logic Puzzles

gets the most correct answers. Feel free to adapt the mental math exercises in any way that will make them more fun and engaging for you.

If you are unable to work with a partner, the mental math exercises can be adapted for solo practice by creating a short list of problems to challenge yourself with. You should write your problem list on a piece of paper. As you work, write down your answers, but **all calculations should be done in your head.** You can then use a calculator to verify your answers. To make things more interesting, you could challenge yourself to beat the clock or to get all answers correct.

This lesson contains two sets of mental math warm-ups:

- Mental Math Set A: The Sums Game and Rounding
- Mental Math Set B: Estimation Games

Complete one set each week.

See the student coursebook for a full description of the mental math activities for each lesson. You are encouraged to partner with your student for these activities. This will show your interest in what your student is learning, allow you to determine areas in which your student is confident and areas that need more work, and let you share in the enjoyment of the challenges and successes of each activity.

Assignments

Textbook Assignments and Test

1. Read textbook sections 1.1 through 1.3 in *Math for Your World* (Blitzer 2016). For each section, follow along with the examples and try the Checkpoint problems. Check your answers against those at the back of the textbook. Verbally answer the Concept and Vocabulary Check exercises at the end of the section. Check your answers with those at the back of the book.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 1.1 through 1.3 (choose from the odd-numbered problems only). Choose several problems of each type to ensure sufficient practice.
3. Do all odd-numbered Application Exercises for Exercise Set 1.2 and 1.3 (no Application Exercises are necessary for Exercise Set 1.1). Check your answers with the back of the book. Make any necessary corrections and review areas that need work. If you need additional practice, you may want to complete a selection of even-numbered problems.
4. Review the Chapter 1 Summary at the end of the chapter. If you feel you need additional practice in any area, select problems from the Chapter 1 Review at the end of the chapter.
5. Complete the Chapter 1 Test from the textbook (for independent students) or the Lesson 1 Test from the test packet (for enrolled students). Students who are using the curriculum independently will complete the test in the textbook and check their answers in the back of the

book. Make necessary corrections and review areas that need work. **Students who are enrolled in Oak Meadow School must complete the Lesson 1 Test from the test packet and submit it to their teacher for grading.**

Math Journal

Complete both journal assignments (do one per week):

- Journal A: Math and You
- Journal B: The Four Step Problem Solving Method in Action

See the student coursebook for full description of the math journaling activities in each lesson. Ask your student to share the math journal at the end of each lesson. This journal is primarily a learning tool for your student, but it can also give you insight into your student's understanding of each topic. When a specific response is required, the journal activity will be included here.

Journal A: This purpose of this journal assignment is to get students thinking about how their past experiences with math have shaped their current attitude toward the subject. This can often prove enlightening for students, enabling them to recognize what they like or dislike about math, and understand why they have developed those attitudes. It can also provide insight into how they best learn math and what has and hasn't worked for them in the past. This journal entry may bring up some past emotional experiences for the student, so there may be some uncomfortable feelings about sharing it. If your student does share it with you, offer encouragement and remind your student about past accomplishments in math—especially times when hard work and perseverance paid off. This journal entry could be a good starting point for a conversation about goals and strategies for this course, but try to keep such discussion light, without criticism or pressure.

Journal B: In assessing this assignment, look for a clearly-defined problem statement and an appropriate response for each of the four steps of the problem-solving method described in the textbook.

Activities

Complete all three activities below.

- Activity A: Modeling College Graduation Rate
- Activity B: Figurate Numbers and Pascal's Triangle
- Activity C: Logic Puzzles

Activities will be included in full in this teacher manual only when a specific response is evident. To see all activities in full, please see the student coursebook.

Activity A: Modeling College Graduation Rate

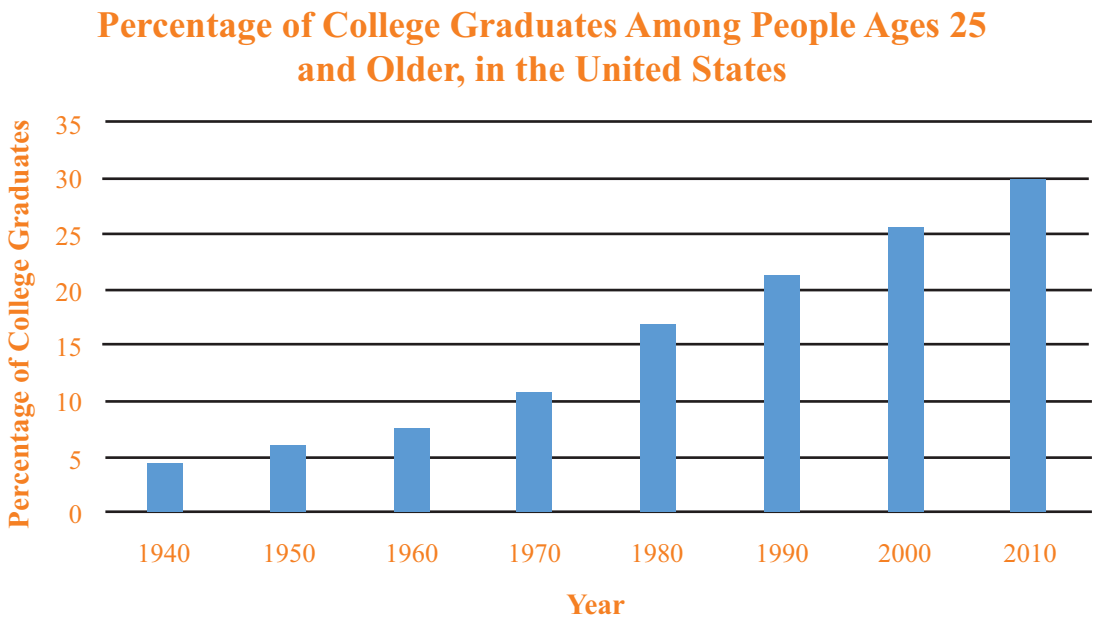
In section 1.2, you explored how to create a linear model for graphed data and use it to make estimates. In this activity, you will create your own bar graph, develop a model that estimates the relationship between the variables, and use your model to make a prediction.

Percentage of College Graduates Among People Ages 25 and Older, in the United States

Year	1940	1950	1960	1970	1980	1990	2000	2010
Percentage	4.6	6.2	7.7	11.0	17.0	21.3	25.6	29.9

Source: U.S. Census Bureau

Make a bar graph for the data in the chart above. Place the years on the horizontal axis and percentage of college graduates on the vertical axis. Be sure to give the graph a title and label the axes. Use a straightedge and graph paper to create neat lines. Tip: Refer to example 8 in section 1.2 for a similar problem.



1. Estimate the increase in the percentage of college graduates per year. Round your answer to the nearest tenth of a percent. Show all of the steps you took to get your answer.

   Answers will vary, depending on data points chosen. One example: $\frac{29.9 - 4.6}{2010 - 1940} = \frac{25.3}{70} = 0.36$, which rounds to 0.4. This indicates that the percentage of college graduates increases by about .4 per year.

2. Now write a mathematical model (an equation) that estimates y, the percentage of Americans age 25 and older who graduated college *x* years after 1940.

   Answers will vary based on the calculation performed in #1. Continuing our example above: $y = 0.4x + 4.6$.

3. Using your model from step 2, project the percentage of college graduates for the year 2020. Again, show your steps.

   Answers will vary based on the equation obtained for #2. Continuing our example: $y = 0.4(80) + 4.6 = 36.6$. The projected percentage of college graduates in the year 2020 is 36.6.

Activity B: Figurate Numbers and Pascal's Triangle

In this activity, we will explore certain "special" numbers called *figurate numbers* and look for patterns.

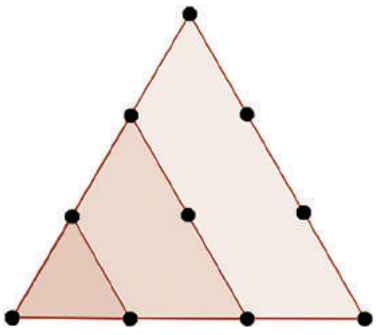
Figurate numbers are special because they can be represented by an arrangement of equally spaced dots in a regular geometrical shape. In other words, they are the number of evenly spaced dots needed to form both the outline and interior of a particular shape. You were introduced to figurate numbers in the Application Exercises for section 1.1. Now we will dig a bit deeper into the patterns formed by these special numbers. This activity has two parts.

Part 1

Important note: The number 1 is defined to be the first figurate number for all shapes, as a single point could form the basis for any shape.

1. If the pattern of the number of dots forms a triangle, then the number of dots is called a **triangular number**. The number 3 is a triangular number because three evenly spaced dots form a triangle. Likewise, the numbers 6 and 10 are also triangular numbers.

   Viewing tip: In the figure below, start by looking at the darkest triangle, which contains three dots. Now look at the triangle formed by overlapping the darkest triangle and the slightly lighter region. This second triangle contains six dots. Finally, look at the largest triangle which contains all the points from the first two triangles. Notice that it contains ten dots, nine on its exterior and one on its interior.



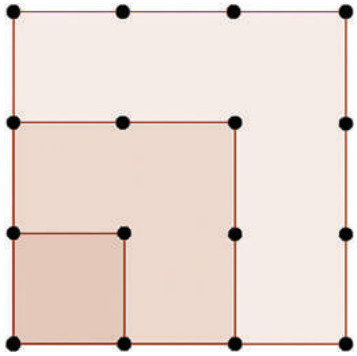
Triangular numbers: 1, 3, 6, 10,...

   Look for a pattern in the first four triangular numbers listed in the caption above. (Hint: note a pattern with the difference between each number and the following number.) Describe your pattern and find the next four triangular numbers. You may find it helpful to extend the figure so that larger triangles are formed. This will allow you to count the dots and verify the pattern.

The pattern shows each triangular number is formed by adding the number of dots in its row to the number of dots in each previous row: 1 = 1, 3 = 1 + 2, 6 = 1 + 2 + 3, 10 = 1 + 2 + 3 + 4, 15 = 1 + 2 + 3 + 4 + 5, and so on. The next four triangular numbers are 15, 21, 28, and 36.

2. A pattern of dots that forms a solid square gives us the **square numbers**.

   Viewing tip: Start by looking at the darkest square in the figure below and notice that it contains four dots. Then look at the medium-sized square that overlaps the smallest square. This contains nine dots. Finally, look at the largest square formed by overlapping the smaller squares, and verify that it contains sixteen dots, twelve on its exterior and four on its interior.

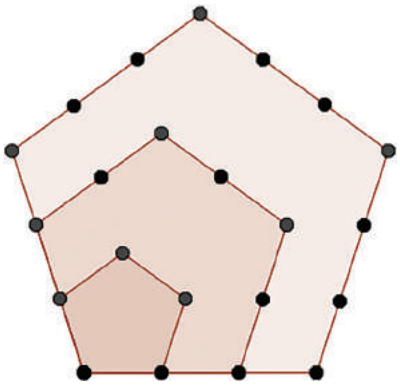


Square numbers: 1, 4, 9, 16,...

   Look for a pattern in the first four square numbers listed in the caption above. Find the next 4 square numbers and describe your pattern.

   In this pattern, each square number is formed by squaring the number of dots in its row: $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, and so on. The next four square numbers are 25, 36, 49, and 64.

3. Similarly, the pentagonal numbers are generated by the pattern of dots forming regular pentagons.

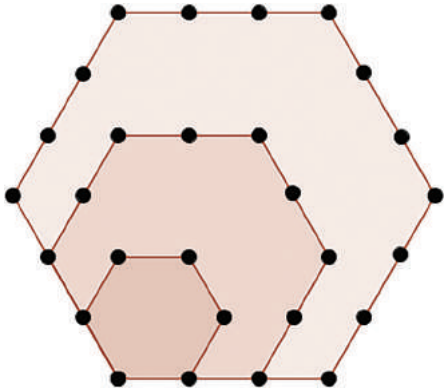


Pentagonal Numbers: 1, 5, 12, 22, . . .

   Look for a pattern in the first four pentagonal numbers listed in the caption above. Find the next 4 pentagonal numbers and describe your pattern.

In looking for this pattern, note that the difference between the first two pentagonal numbers is 4, then the difference between the second and third pentagonal numbers is 7, then the next difference is 10. These differences increase by 3 each time. Following this pattern, we expect the next difference to be 13, and the following difference to be 16, and so on. The next four pentagonal numbers are 35, 51, 70, and 92.

4. And the pattern of dots forming regular hexagons give us the hexagonal numbers.



Hexagonal Numbers: 1, 6, 15, 28, . . .

Look for a pattern in the first four hexagonal numbers listed in the caption above. Find the next 4 hexagonal numbers and describe your pattern.

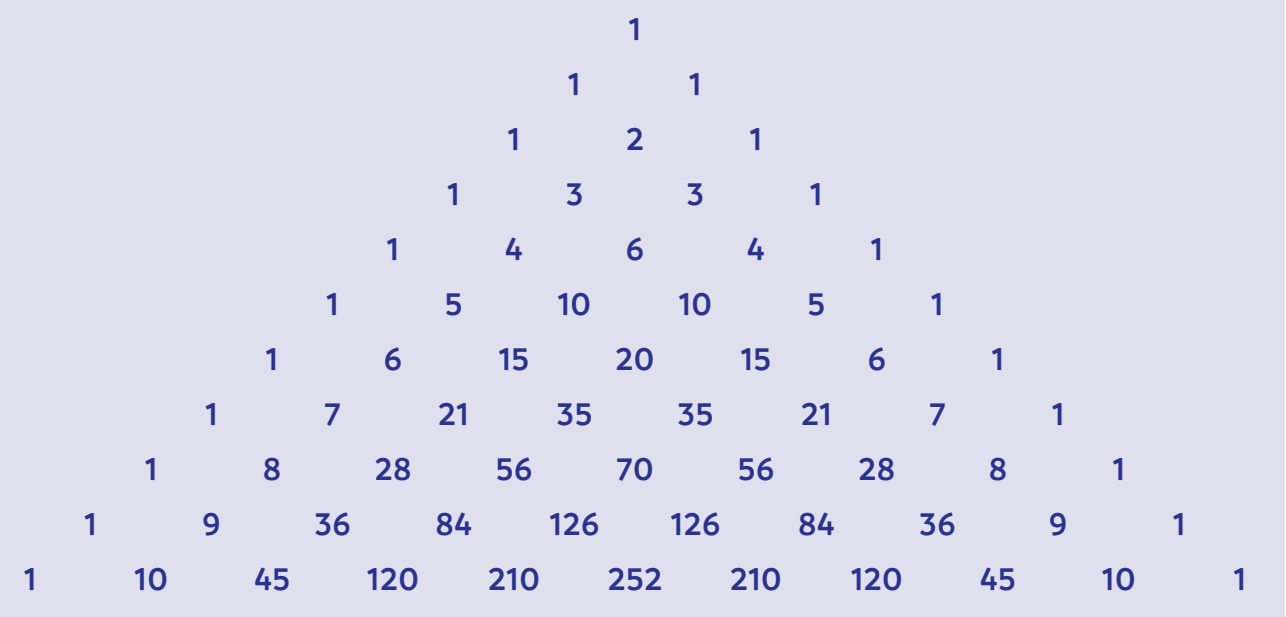
This pattern is similar to the pentagonal number pattern, but note the difference between each listed hexagonal number and the next one. The differences are 5, 9, and 13. There is an increase of 4 for each difference. Following this pattern, we expect the next difference to be 17, then 21, and so on. The next four hexagonal numbers are 45, 66, 91, and 120. (Another pattern is that every other triangular number is a hexagonal number.)

Part 2

Once you have done that, you are ready to explore more patterns with the figurate numbers using **Pascal's Triangle**. Pascal's Triangle is a triangular arrangement of numbers that was named for mathematician Blaise Pascal (1623–1662), who studied it extensively. (We'll learn more about Blaise Pascal in lesson 11.) The triangle itself was discovered long before Pascal was even born, though; a version from 13th century China has been found!

The arithmetic triangle, as Pascal's Triangle is also known, is constructed by placing the number one at the top of a page with two ones in the row beneath it. Each successive row is formed with one more entry than the row above it, a 1 at the very left-most entry and the very right-most entry. Each of the remaining entries is the sum of the two numbers directly above it. (See the diagram below.)

Pascal's Triangle has many interesting properties, enough to take up an entire course! For now, what we are interested in is the patterns formed by the figurate numbers in Pascal's Triangle. Your mission is to locate some of them.



The first 11 rows of Pascal's Triangle

1. The diagram above shows the first 11 rows of Pascal's Triangle. Following the pattern, write down the 12th row.

   12th row of Pascal's Triangle: 1 11 55 165 330 462 462 330 165 55 11 1

2. The triangular numbers are the easiest to find in Pascal's Triangle. Using your list of triangular numbers from Part 1, look for them in the triangle. Use a **red** pen or pencil to circle the triangular numbers on the diagram. (If you need a hint, look at the diagonals.) Describe the pattern in Pascal's Triangle for triangular numbers. Does your pattern verify the next four triangular numbers you identified in Part 1, step 1?

   The 2nd diagonals should have all entries circled in red.

3. The hexagonal numbers are in a similar location in Pascal's Triangle. Using your list of hexagonal numbers from Part 1, look for them in the triangle. Use a **blue** pen or pencil to circle the hexagonal numbers on the diagram, making sure to also leave previous markings visible. Describe the pattern in Pascal's Triangle for hexagonal numbers. Does your pattern verify the next four hexagonal numbers you identified in Part 1, step 4?

   Every other entry in the 2nd diagonals should be circled in blue.

4. The square numbers and pentagonal numbers can also be found on Pascal's Triangle, but they are not quite as obvious because they require adding entries together in the triangle. To see how these work, and to learn about other applications of Pascal's Triangle to patterns of numbers, visit http://www.mathsisfun.com/pascals-triangle.html. (This link, along with all the other links mentioned in this course, can be found in clickable form on the Math Connections resource page on the Oak Meadow website at www.oakmeadow.com/curriculum-links/. Bookmark that page for easy access to all the recommended online sources!)

Lesson

2 Set Theory

This lesson should take approximately three weeks to complete.

Learning Objectives

- Represent a set using a description, the roster method, and set-builder notation.
- Distinguish between finite and infinite sets.
- Recognize subsets and the empty set, and use appropriate notation.
- Represent set relationships using Venn diagrams.
- Find the complement of a set, and the intersection and union of two sets.
- Perform operations with sets.
- Use Venn diagrams to solve problems.
- Apply knowledge of set operations and Venn diagrams to create and solve problems.
- Identify a mathematician of personal interest.

Mental Math Warms-ups

This lesson contains three sets of mental math warm-ups. Complete one set each week.

- Mental Math Set A: Grouping Strategies
- Mental Math Set B: Multiplication Strategies
- Mental Math Set C: Division Strategies

ASSIGNMENT SUMMARY

- ☐ Mental Math Set A: Grouping Strategies
- ☐ Mental Math Set B: Multiplication Strategies
- ☐ Mental Math Set C: Division Strategies
- ☐ Read Chapter 2 in textbook.
- ☐ Complete a selection of exercises for sections 2.1 through 2.5.
- ☐ Read Chapter 2 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: How Big is Infinity?
- ☐ Math Journal B: Activity Reflection
- ☐ Complete mathematician project proposal.
- ☐ Activity A: Blood Types and Venn Diagrams
- ☐ Activity B: Create Your Own Survey Problem

Assignments

Textbook Assignments and Test

1. Read textbook sections 2.1 through 2.5 in Math for Your World. For each section, follow along with the examples and try the Checkpoint problems. Check your answers with the back of the book. Verbally answer the Concept and Vocabulary Check exercises at the end of the section. Check your answers with the back of the book.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 2.1 through 2.5 (odd-numbered problems only). Choose several problems of each type to ensure sufficient practice.
3. Do all odd-numbered Application Exercises for each Exercise Set. Check your answers with the back of the book. Make any necessary corrections and review areas that need work. If you need additional practice, you may want to complete a selection of even-numbered problems.
4. Review the Chapter 2 Summary at the end of the chapter. If you feel you need additional practice, select problems from the Chapter 2 Review at the end of the chapter.
5. Complete the Chapter 2 Test from the textbook (for independent students) or the Lesson 2 Test from the test packet (for enrolled students). Students who complete the textbook test are encouraged to check their answers in the back of the book, making necessary corrections and reviewing areas that need work. **Students who are enrolled in Oak Meadow School must complete the test from the test packet.**

Math Journal

Complete both journal assignments. Tip: do one per week.

- Journal A: How Big is Infinity?
- Journal B: Activity Reflection

Project Milestone

Mathematician Project Topic Proposal

After completing lesson 6, you will wrap up the first semester by writing a paper for your midterm project. This week, you will begin working on your project by creating a proposal.

For your midterm project, you will write a 3–5 page research-based paper on a mathematician of your choice. But there's a twist: the paper must be written from the point of view of your chosen mathematician, as if he or she were writing an autobiography. This will allow you to "get into the head" of your mathematician, like a character actor, and relive his or her life and mathematical discoveries. While the paper must be grounded in fact and will require citations, you have full creative license to interpret the circumstances surrounding the facts and imagine your mathematician's thoughts and feelings. You

may choose to write the paper in the form of a letter, diary entries, a memoir, an interview, or even an obituary "written" by your mathematician. Be creative!

This paper should focus on the mathematical work done by your chosen person. Information such as the early life and education is important, but the spotlight should be on the person's mathematical ideas. You are not expected to fully understand this person's work, but you should be able to explain it in general terms.

Since this is a large project, it is best approached by researching, organizing, and writing the paper in stages over the course of the semester, culminating with the final submission after lesson 6. There will be three "milestone assignments" along the way: the topic proposal, the bibliography, and the outline.

In this lesson, you will accomplish the first milestone: choosing a mathematician to write about. The list below contains some popular suggestions, but you are free to select any mathematician, even one not listed. For inspiration, flip through this coursebook and look at the "People in Mathematics" spotlights found in each lesson. There is also an extensive index of mathematicians on the MacTutor History of Mathematics Archive at http://www-history.mcs.st-and.ac.uk/BiogIndex.html.

You are encouraged to choose a mathematician who is new to you, but please make sure you can find sufficient information to write a 3–5 page paper. If you are up for a challenge, choose a mathematician who is female, or from another culture, or someone who is alive and working today.

For this lesson's milestone, select your mathematician and do some preliminary research. This will let you get to know your mathematician and assess whether enough information is available for you to write a paper about that person. Write a two-paragraph topic proposal that includes:

- The mathematician's name and a sentence or two describing this person's mathematical contributions in your own words.
- An explanation of why you selected this person.
- A citation of the source(s) of your preliminary research information.

Important reminder for all students: Use only reputable sources written by an authority on the subject. The MacTutor History of Mathematics Archive mentioned above is an excellent starting point. Reputable encyclopedias and dictionaries of scientific biography are fine, too. Please note that sites like Wikipedia are never acceptable sources for research information. In most cases, student and teacher webpages are not acceptable, either. Also avoid blog posts (unless written by an authority on the subject) and popular TV station sites like bio.com. If you have a question about whether a source is reputable, ask your parent, tutor, or teacher.

Some suggested mathematicians (remember, you are not limited to people on this list):

Muhammad al-Kwarizmi
Archimedes
George Boole
Georg Cantor
Leonardo DaVinci
Charles Dodgson (aka Lewis Carroll)
René Descartes
Albert Einstein
Paul Erdös
Leonhard Euler
Pierre de Fermat
Leonardo de Pisa (Fibonacci)
Leonardo da Vinci
Galileo Galilee
Carl Friedrich Gauss
Sophie Germain
Stephen Hawking
Omar Khayyám
Ada Lovelace
John Nash
Isaac Newton
Emmy Noether
Blaise Pascal
Plato
Pythagoras
George Pólya
Srinivasa Ramanujan
Alan Turing

Activities

Complete the activities below.

- Activity A: Blood Types and Venn Diagrams
- Activity B: Create Your Own Survey Problem

Activity A: Blood Types and Venn Diagrams

The discovery and classification of different antigens in blood revolutionized the field of medicine. Re-read the information about blood types in the Blitzer Bonus box on page 91 of *Math for Your World* and refer to the Venn diagram in Figure 2.23. Note, in particular, that the Venn diagram classifies the eight different blood types by the presence or absence of each of the three antigens A, B, and Rh in red blood cells. Also note that in order to receive blood in a transfusion, "the recipient must have all or more of the antigens present in the donor's blood." This means that "the set of antigens in a donor's blood must be a subset of the set of antigens in a recipient's blood." (Blitzer, 95)

Let's consider, for example, that Marina, who has type A+ blood, needs a blood transfusion. To have type A+ blood, Marina must have antigens A and Rh, but not B, present in her blood. Because she needs to have all or more of the antigens in the donor's blood, Marina cannot receive blood that contains B antigens. Looking at the Venn diagram, we see that this limits Marina to receiving blood only from donors with O+, O–, A+, or A– blood type.

Use the information provided on page 91 to fill in the chart below and then answer the questions that follow.

Recipient Blood Types	Compatible Donor Blood Types
A+	O+, O–, A+, A–
B+	**O+, O–, B+, B–**
AB+	**O+, O–, A+, A–, B+, B–, AB–, AB+**
O+	**O+, O–**
A–	**A–, O–**
B–	**B–, O–**
AB–	**AB–, A–, B–, O–**
O–	**O–**

1. A universal recipient is a person who can receive blood from a donor with any blood type. Based on the Venn diagram and your chart above, which blood type does a universal recipient have?

   AB+

2. A universal donor is a person who can donate blood to a person with any blood type. Based on the Venn diagram and your chart above, which blood type does a universal donor have?

   O–

3. Nikki, who has blood type B–, is donating blood. What blood type(s) must a recipient be to receive her blood?

   B+, B–, AB+, or AB–

4. Miguel was in a serious accident and needs a blood transfusion. A quick test at the ER lab indicates that he has O+ blood. What donor blood type(s) can he receive?

   O+ or O–

Activity B: Create Your Own Survey Problem

Now it's your turn! Make up your own three-set survey problem about a fictional situation of your choice. Note: this is fictional, so you will not actually conduct a survey.

For the sake of simplicity, assume that you will survey 100 fictional participants. Start by creating a three-set Venn diagram and fill in fictional numbers so that all 100 participants are represented. Be sure to consider including participants in the region outside of the three sets, and don't forget that the intersections of the sets are included in the total number for each set. Remember, the sum of the numbers in all of the individual regions on the Venn diagram should be 100.

The next step is to determine how much information you must present in order to make the problem solvable. What questions will you ask? For inspiration, refer to similar problems in the Application

Exercises on pages 103–104 of the textbook, but be sure that you create a unique problem that will require the solver to make a three-set Venn diagram.

Include at least three questions pertaining to your problem. Also provide the solution to your problem, including a completed and labeled Venn diagram illustrating your problem. You may use or copy the empty Venn diagram below.

If possible, briefly explain to a friend or family member how to solve survey problems and have them test out your problem. This will help you determine if you provided enough information to create a Venn diagram and answer your questions.

Answers will vary, but in evaluating the student-created problem, make sure that it is solvable and that the Venn diagram supports the solution. The numbers in each section of the Venn diagram, including the part outside the rings, should sum to 100. A common mistake made by students is to not subtract the numbers in the intersections that should have already been counted if starting from the center of the diagram.

Lesson

3

Number Theory and the Real Number System

This lesson should take approximately three weeks to complete.

Learning Objectives

- Determine whether a natural number is prime or composite.
- Apply divisibility rules to natural numbers.
- Find the greatest common factor and prime factorization for a number.
- Find the least common multiple for two numbers.
- Apply the order of operations.
- Perform operations and solve problems with rational numbers.
- Simplify and perform operations with square roots.
- Classify numbers into sets and subsets.
- Convert between decimal and scientific notation.
- Write terms of arithmetic and geometric sequences.
- Explore recent mathematical discoveries of prime numbers.
- Identify prime numbers and recognize patterns using the Sieve of Eratosthenes.
- Explore some real-world applications of the Fibonacci sequence.

Mental Math Warms-ups

This lesson contains three sets of mental math warm-ups. Complete one set each week.

- Mental Math Set A: Divisibility Rules
- Mental Math Set B: Exponents
- Mental Math Set C: Scientific Notation

ASSIGNMENT SUMMARY

- ☐ Mental Math Set A: Divisibility Rules
- ☐ Mental Math Set B: Exponents
- ☐ Mental Math Set C: Scientific Notation
- ☐ Read Chapter 3 in textbook.
- ☐ Complete a selection of exercises for sections 3.1 through 3.7.
- ☐ Read Chapter 3 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: Divisibility Trick for Seven
- ☐ Math Journal B: The Largest Known Prime Number (so far)
- ☐ Math Journal C: Fibonacci Connections
- ☐ Activity A: The Sieve of Eratosthenes
- ☐ Activity B: Fibonacci Inspiration

Assignments

Textbook Assignments and Test

1. Read textbook sections 3.1 through 3.7. For each section, follow along with the examples and try the Checkpoint problems. Check your answers with the back of the book. Verbally answer the Concept and Vocabulary Check exercises at the end of the section and check your answers with the back of the book.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 3.1 through 3.7 (odd-numbered problems only). Choose several problems of each type to ensure sufficient practice.
3. Do the following Application Exercises:
    - Exercise Set 3.1: Application Exercises 91 and 95.
    - Exercise Set 3.2: Application Exercises 115, 127, and 129.
    - Exercise Set 3.3: Application Exercises 117, 119, 121, 123, 127, 131, and 133.
    - Exercise Set 3.4: Application Exercises 75, 77, and 79.
    - Exercise Sets 3.5 through 3.7: do all Application Exercises (odds only).

    Check your answers with the back of the book. Make any necessary corrections and review areas that need work. Feel free to complete a selection of even-numbered problems for extra practice.
4. Review the Chapter 3 Summary at the end of the chapter. Select problems from the Chapter 3 Review at the end of the chapter if you feel you need additional practice.
5. Complete the Chapter 3 Test from the textbook (for independent students) or the Lesson 3 Test from the test packet (for enrolled students). Students who complete the textbook test can check their answers in the back of the book, making necessary corrections and reviewing areas that need work. **Students who are enrolled in Oak Meadow School must complete the test from the test packet.**

Math Journal

Complete all three journal assignments (do one per week).

- Math Journal A: Divisibility Trick for Seven
- Math Journal B: The Largest Known Prime Number (so far)
- Math Journal C: Fibonacci Connections

Activities

Complete both activities below.

- Activity A: The Sieve of Eratosthenes
- Activity B: Fibonacci Inspiration

Activity A: The Sieve of Eratosthenes

You have already learned that a prime number is a natural number greater than 1 that has exactly two natural number factors, namely 1 and itself. You also learned that a composite number is a natural number greater than 1 that has more than two natural number factors.

Suppose we want to find all of the prime numbers between 1 and 100. We could examine each number between 1 and 100, trying to find factors as we did in Section 3.1, but that would be very tedious and time-consuming. Good news: there is a better way!

The Greek mathematician Eratosthenes came up with a method of finding prime numbers, which we call today the Sieve of Eratosthenes. A sieve is a strainer or sifter that lets small items pass through while catching big items, just like a kitchen colander. In this case, the sieve is filtering out (crossing out) composite numbers and keeping (circling) prime numbers. Eratosthenes' method uses a chart of numbers and the instructions below. Follow the instructions, crossing out and circling numbers on the chart as indicated, and then answer the questions.

Step 1: Cross out the number 1. (Remember, 1 is neither prime nor composite!)

Step 2: Circle 2 because it is a prime number. Then cross out every multiple of 2 on the chart. (Why? Because any number that is a multiple of 2 has 2 as a factor and is, therefore, composite.) This eliminates all of the even numbers except for 2.

Step 3: Circle 3 because it is the next prime number. Then cross out every multiple of 3 on the chart, as these numbers will also be composite.

Step 4: Circle 5 because it is the next prime number. Then cross out every multiple of 5 on the chart.

Step 5: Continue in this way (circle 7 and cross out its multiples, and so on) until every number from 1 to 100 has either been circled or crossed out. The circled numbers are all the prime numbers from 1 to 100.

After you have done that, answer the following questions.

1. Why did the instructions have you skip checking the number 4 and its multiples?

   Every multiple of 4 was already crossed out in the step to check multiples of 2.

2. List all of the prime numbers between 1 and 100. How many primes are there between 1 and 100?

   There are 25 prime numbers between 1 and 100: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.

3. When exploring mathematics, we always look for patterns. One pattern you may have noticed is that for each number whose multiples you crossed out, you did not have to cross out any multiples until you reached the square of that number. For example, when you crossed out multiples of 3, you circled 3 because it was prime, but 6 was already crossed out. $3^2 = 9$ was the first multiple that needed to be crossed out. Likewise, when you crossed out multiples of 5, the multiples 10, 15, and 20 were already crossed out, leaving $5^2 = 25$ as the first multiple to be crossed out. Why is this the case? (Hint: think about what numbers are divisors of the numbers already crossed out.)

   Because all numbers less than itself that were multiplied by that number have already been checked. In the case of 5, we already checked 2, 3, and 4, so 5×2, 5×3, and 5×4 were already crossed out. That leaves 5×5 *or* 5^2 as the first multiple of 5 that we actually need to check.

4. What was the largest prime number from 1 to 100 that had a multiple that still needed to be crossed out? In other words, after which prime number could you stop checking for multiples? How did you know for certain that there would be no more primes between 1 and 100? Try to base your explanation on the pattern mentioned in question 3.

   7 is the last prime number that needs to be checked for multiples because it is the largest prime number less than 10 (since $\sqrt{100} = 10$). Once we reach the next prime number, 11, its multiples that need to be checked (beginning with 11^2) are greater than 100 and will not appear on this chart.

5. Suppose you were asked to use the Sieve of Eratosthenes to find all prime numbers between 1 and 400. What would be the largest prime number you would need to check before you could stop crossing out composite numbers and declare all prime numbers found?

   19 is the last prime number that needs to be checked for multiples because it is the largest prime number less than 20 (since $\sqrt{400} = 20$). Once we reach the next prime number, 23, its multiples that need to be checked (beginning with 23^2) are greater than 400 and will not appear on the chart to up to 400.

6. Apply what you have learned through this activity to find the next five prime numbers greater than 100.

   101, 103, 107, 109, and 113

7. An *emirp* (prime written backward) is a prime number whose digits can be reversed and form a different prime number. The number 13 is the smallest emirp. The reverse of its digits is 31, which is also a prime number. What other emirps do you see on your chart of prime numbers?

   17 and 71 are emirps. So are 37 and 73, as well as 79 and 97.

8. If a prime number is doubled and increased by one, and the result is a prime number, then the result $2p + 1$ is called a Sophie Germain prime. For example, 11 is a Sophie Germain prime because it is the result when the prime number 5 is doubled and increased by 1, but the prime number 19

is not a Sophie Germain prime because 19 = 2(9) + 1 and 9 is not prime. Select any three prime numbers under 100 and test to see if they are Sophie Germain primes. Show your steps.

Answers will vary. Check that the numbers are tested correctly.

Lesson

4

Algebra: Equations and Inequalities

This lesson should take approximately three weeks to complete.

Learning Objectives

- Evaluate and simplify algebraic equations.
- Use mathematical models to solve problems.
- Solve a formula for a specified variable.
- Compute percent discounts and increases.
- Solve problems using proportions.
- Solve problems involving direct or indirect variation.
- Solve problems involving linear inequalities.
- Locate reputable sources on a chosen mathematician.
- Evaluate the validity of a given solution to a problem.
- Design original problems to be solved using linear equations and inequalities.
- Consider the nature of mathematics.

Mental Math Warms-Ups

This lesson contains three sets of mental math warm-ups. Complete one set each week.

- Mental Math Set A: Percentages
- Mental Math Set B: Algebraic Translation
- Mental Math Set C: Beat the Calculator

ASSIGNMENT SUMMARY

- ☐ Mental Math Set A: Percentages
- ☐ Mental Math Set B: Algebraic Translation
- ☐ Mental Math Set C: Beat the Calculator
- ☐ Read Chapter 4 in textbook.
- ☐ Complete a selection of exercises for sections 4.1 through 4.6.
- ☐ Read Chapter 4 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: Percentage Problem OR Math Journal B: Is This Correct?
- ☐ Complete mathematician project bibliography.
- ☐ Activity A: Create Your Own Word Problems
- ☐ Activity B: Is Mathematics Invented or Discovered?
- ☐ Activity C (optional): Diophantus' Riddle

Assignments

Textbook Assignments and Test

1. Read textbook sections 4.1 through 4.6.For each section, follow along with the examples and try the Checkpoint problems. Check your answers with the back of the book. Verbally answer the Concept and Vocabulary Check exercises and check your answers.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 4.1 through 4.6 (odd-numbered problems only). Choose several problems of each type to ensure sufficient practice.
3. Do the following Application Exercises:
   - Exercise Set 4.6: Application Exercises 95–101 (odd-numbered problems only).
   - All other Exercise Sets: do all Application Exercises (odds only).

   Check your answers and make any necessary corrections. Review areas that need work.
4. Review the Chapter 4 Summary at the end of the chapter. If you feel you need additional practice, select problems from the Chapter 4 Review at the end of the chapter.
5. Complete the Chapter 4 Test from the textbook (for independent students) or the Lesson 4 Test from the test packet (for enrolled students). Students who complete the textbook test are encouraged to check their answers in the back of the book, making necessary corrections and reviewing areas that need work. **Students who are enrolled in Oak Meadow School must complete the test from the test packet.**

Math Journal

Choose one of the following math journal assignments (either Journal Assignment A or Journal Assignment B). Be sure to indicate which assignment you chose.

- Math Journal A: Percentage Problem
- Math Journal B: Is This Correct?

Journal A: Percentage Problem

In January of last year, Joe resolved to eat well and exercise, and he lost 10% of his body weight. This year he did not keep up with his healthy habits and gained 10% of his body weight. Does Joe now weigh the same amount he did last January? Why or why not? Give an example weight for Joe and demonstrate your calculations. Be sure to label Joe's weight at the start and end of each year.

Joe does not weigh the same as he did last January. Examples will vary, but here is one sample:

Last year's starting weight: 200 lb. After losing 10% of his body weight, Joe weighed $200 - (200 \times 0.10) = 180$ lb at the end of the first year, and therefore at the start of the second

year. During the second year, he gained back 10% of his body weight: 180 + (180 × 0.10) = 198 lb. He now weighs 198 lb.

Journal B: Is It Correct?

Suppose your friend, Brandon, asks you to check over his math homework. You read his solution to the first problem:

$$-2(x-4) > x+7$$

$$-2x-8 > x+7 \qquad -2x+8 > x+7$$

$$-3x-8 > 7 \qquad -3x+8 > 7$$

$$-3x > 15 \qquad -3x > -1$$

$$x > -5 \qquad x < 1/3$$

Is Brandon's solution correct? If not, point out any mistakes and explain to Brandon where he went wrong and what he should do instead to correctly solve the inequality.

Mistake #1: Brandon did not distribute the –2 to the –4 in his first step. He got –8 when the product should have been 8. This caused him to get 15 instead of –1 on the right side of the equation in step 3.

Mistake #2: Brandon also forgot that when multiplying or dividing an inequality by a negative number, the inequality sign flips.

Project Milestone

Mathematician Project Bibliography

As the next milestone for your midterm project, locate at least three reputable sources for your chosen mathematician and create a preliminary bibliography. At least two of your sources must consist of books, journal articles, encyclopedias, etc. (Digital copies of books and articles published originally in print form are acceptable.) Additional sources may include materials found solely online.

Refer to the instructions in the appendix in this course book to learn more about reputable sources and how to create bibliography entries in MLA format. You should also refer to the midterm project instructions in lesson 2 for general information on the project's requirements and a few special notes on acceptable sources.

A dictionary of scientific biography, which can typically be found in a library's reference section, is a great choice of source. Articles in these books are generally concise and reliable.

Activities

Complete activities A and B below. Activity C is optional. (Enrolled students will earn extra credit if they complete Activity C).

- Activity A: Create Your Own Word Problems
- Activity B: Is Mathematics Invented or Discovered?
- Activity C (optional): Diophantus' Riddle

Activity C (optional): Diophantus' Riddle

The Ancient Greek mathematician Diophantus was the subject of a 5th century algebra problem written in the form of an epitaph:

> Here lies Diophantus, the wonder behold.
>
> Through art algebraic, the stone tells how old:
>
> God gave him his boyhood one-sixth of his life,
>
> One twelfth more as youth while whiskers grew rife;
>
> And then yet one-seventh ere marriage begun;
>
> In five years there came a bouncing new son.
>
> Alas, the dear child of master and sage
>
> After attaining half the measure of his father's life chill fate took him.
>
> After consoling his fate by the science of numbers for four years, he ended his life.

How old was Diophantus at the time of his death? Begin by defining your variable and writing an algebraic expression to represent the situation. Solve the equation, showing all steps.

x = **Diophantus's age at his death**

$$x = \frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \frac{1}{2}x + 4$$

$$x = \frac{14}{84}x + \frac{7}{84}x + \frac{12}{84}x + 5 + \frac{42}{84}x + 4$$

$$x = \frac{75}{84}x + 9$$

$$84x = 75x + 756$$

$$9x = 756$$

$$x = 84 \text{ years}$$

Lesson

5

Algebra: Graphs, Functions, Linear Functions, and Linear Systems

This lesson should take approximately two weeks to complete.

Learning Objectives

- Graph equations and functions on the coordinate plane.
- Identify characteristics of a function based on its graph.
- Calculate and interpret slope.
- Model data using linear equations.
- Create a scatterplot and interpret information from it.
- Solve linear equations by graphing, substitution, and addition.
- Solve problems using systems of linear equations.
- Graph a linear equation in two variables.
- Graph a system of linear inequalities.
- Use mathematical models involving linear inequalities.
- Organize project research notes into an outline.
- Solve application problems.

Mental Math Warms-ups

This lesson contains two sets of mental math warm-ups. Complete one set each week.

- Mental Math Set A: Visualizing Graphs of Linear Equations
- Mental Math Set B: Correlation Coefficients and Slopes of Lines

ASSIGNMENT SUMMARY

- ☐ Mental Math Set A: Visualizing Graphs of Linear Equations
- ☐ Mental Math Set B: Correlation Coefficients and Slopes of Lines
- ☐ Read Chapter 5 in textbook.
- ☐ Complete a selection of exercises for sections 5.1 through 5.5.
- ☐ Read Chapter 5 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: Correlation
- ☐ Complete mathematician project outline.
- ☐ Activity A: Celsius and Fahrenheit
- ☐ Activity B: Literacy and Hunger Statistics

Assignments

Textbook Assignments and Test

1. Read textbook sections 5.1 through 5.5. For each section, follow along with the examples and try the Checkpoint problems, checking your answers. Verbally answer the Concept and Vocabulary Check exercises and check your answers.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 5.1 through 5.5 (odd-numbered problems only). Choose several problems of each type to ensure sufficient practice.
3. Do the following Application Exercises:
   - Exercise Set 5.4: Application Exercises 51–61 (odds only).
   - All other Exercise Sets: do all Application Exercises (odds only).

   Check your answers and make any necessary corrections. Review areas that need work.
4. Review the Chapter 5 Summary at the end of the chapter. Use the Chapter 5 Review for additional practice, if necessary.
5. Complete the Chapter 5 Test from the textbook (for independent students) or the Lesson 5 Test from the test packet (for enrolled students). Students who complete the textbook test are encouraged to check their answers in the back of the book, making necessary corrections and reviewing areas that need work. **Students who are enrolled in Oak Meadow School must complete the test from the test packet.**

Math Journal

Complete the following journal assignment.

Journal A: Correlation

Answers will vary. Look for an example of a situation that makes sense. Here is one example: The cost of damage done by a fire and the number of firefighters on the scene of the fire are positively correlated. Fires with a lot of firefighters on the scene tend to have higher costs of damage. However, the cost of the damage does not cause the number of firefighters who arrive on the scene, and the number of firefighters on the scene does not the cause the cost of damage done by the fire. These two variables are correlated, but neither causes the other, so causation does not apply. (The *severity* of the fire affects both of these variables. A more severe fire will require more firefighters on the scene. A more severe fire is also likely to have a high cost of damage.)

Project Milestone

Organize your research notes in the form of a 1–2 page outline. A typical outline will likely include such headings as:

I. Introduction

II. Early Life

III. Work (There may be more than one section for different periods or types of work)

IV. Later life

V. Conclusion

If you prefer to use a different format or another form of outlining, such as concept mapping, feel free to do so.

Once you have your major headings, fill in topics and details that you want to cover. The outline need not include every detail you will mention, but it should include all major topics and subtopics from the introduction through the conclusion. The more details you fill in now, the easier it will be to write your rough draft.

Please refer to the midterm project instructions in Lesson 2 for the project's general requirements.

Activities

Complete both of the activities below.

- Activity A: Celsius and Fahrenheit
- Activity B: Literacy and Hunger Statistics

Activity A: Celsius and Fahrenheit

Complete Critical Thinking Exercise #68 on page 309 of the textbook. Show all of your steps.

$$F = mC + b$$

$$m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$$

$$F = \frac{9}{5}C + b$$

$$32 = \frac{9}{5}(0) + b$$

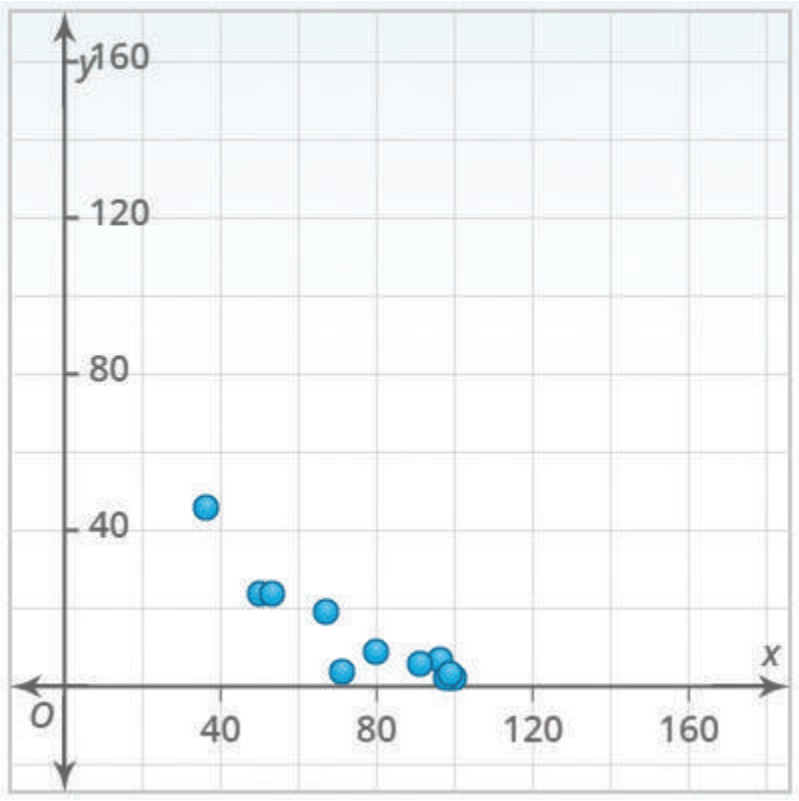
$$b = 32$$

$$F = \frac{9}{5}C + 32$$

Activity B: Literacy and Hunger Statistics

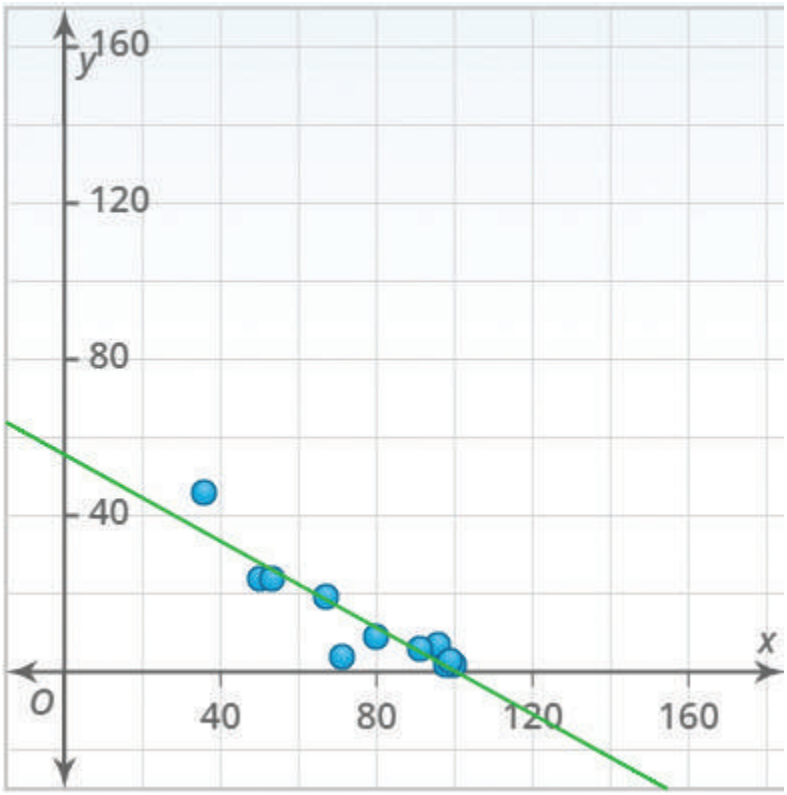
Complete Technology Exercise #60 on page 320 of the textbook. If you plan to use your calculator, simply follow the instructions in the book.

a.



b. $r = -0.9$

c.



d. r = –0.91

***y* = –0.55*x* + 55.62**

Yes, the value of r is consistent with part b.

e. *y* = –0.55(60) + 55.62 = 22.62

We would expect that 22.62 percent of people are undernourished.