

Geometry

Coursebook



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Table of Contents

Introduction	v
Course Materials	v
Course Organization	vi
Academic Expectations	ix
A Note About the Workload	ix
 Lesson 1 A Beginning of Geometry	
Part 1	1
Part 2	7
 Lesson 2 Introduction to Reasoning and Proofs	
Part 1	13
Part 2	21
 Lesson 3 Parallel and Perpendicular Lines	
Part 1	27
Part 2	33
 Lesson 4 Triangles and Congruence	
Part 1	39
Part 2	45
 Lesson 5 Special Properties of Triangles	
Part 1	53
Part 2	59

Lesson 6 Quadrilaterals

Part 1..... 65

Part 2..... 71

Lesson 7 Similarity

Part 1..... 77

Part 2..... 83

Lesson 8 Transformations

Part 1..... 91

Part 2..... 95

Lesson 9 Right Triangles and Trigonometry

Part 1..... 101

Part 2..... 109

Lesson 10 Area

Part 1..... 117

Part 2..... 123

Lesson 11 Surface Area and Volume

Part 1..... 129

Part 2..... 135

Lesson 12 Circles and Other Conic Sections

Part 1..... 141

Part 2..... 147

Appendix

Bonus Unit..... 154

Answer Key to Exploratory Activities..... 155

Answer Key to Even-Numbered Problems in Exercise Sets..... 156

Academic Expectations..... 158

Works Cited..... 159



Introduction

Welcome to *Geometry*!

This course is designed for independent learning, so hopefully you will find it easy to navigate.

However, it is assumed you will have an adult (such as a parent, tutor, or school-based teacher) supervising your work and providing support and feedback. We'll refer to this person as “your teacher” throughout this course. If you have a question about your work, ask your teacher for help.

Please read this entire course introduction before beginning lesson 1. This information will help you be more successful and get the most out of this course.

Course Materials

The following textbook is required for this course:

- *Geometry* by Elayn Martin-Gay (Pearson, 2016)

This textbook is accompanied by an online resource called MyMathLab (MML) that contains many helpful tools, such as instructional videos, chapter test prep videos, a detailed solution manual, and the digital textbook. Using this resource is highly recommended. (Please note: The textbook refers to this resource as MyMathLab, but the website is now called MyLab Math. To match the textbook, in this course we'll use MyMathLab or MML to refer to this online resource.)

Note: MyMathLab is available for individual purchase for students using the course independently. Students who are enrolled can talk to their teacher about how to access MML.

This course draws from a wide variety of online resources, all of which can be accessed through the Oak Meadow website at the following link:

“Curriculum Links”

oakmeadow.com/curriculum-links

Bookmark this page for easy access to all the online resources mentioned in the activities.

You will have the opportunity to use GeoGebra, a dynamic geometry software, for many activities throughout this course. View the introductory video below to familiarize yourself with how to use GeoGebra tools:

“GeoGebra Tutorial”

(You can access this link—along with all the online resources in this course—at oakmeadow.com/curriculum-links.)

Course Organization

This course is organized into 12 lessons that correspond to the chapters in the textbook. Each lesson is divided into two parts, which allows you to submit work to your teacher and get regular feedback.

In addition, there is one optional bonus unit included after the last lesson. If time allows, you are encouraged to complete part or all of this unit for extra enrichment at the end of the course. Please consult with your teacher before beginning the bonus unit.

When you begin each lesson, take a few minutes to look over all the assignments and activity options. This will help you plan your time accordingly. Use the assignment checklist at the beginning of each lesson to check off tasks as you complete them so you can see at a glance what you still need to do.

The length of each chapter varies. Most will take approximately three weeks, but some are shorter. Following the schedule suggested below will allow you to successfully complete the course within a ten-month school year.

Lesson and Textbook Chapter Title	Time to Complete (weeks)
1. A Beginning of Geometry	3
2. Introduction to Reasoning and Proofs	3
3. Parallel and Perpendicular Lines	3
4. Triangles and Congruence	3
5. Special Properties of Triangles	2–3
6. Quadrilaterals	2–3
7. Similarity	3
8. Transformations	3
9. Right Triangles and Trigonometry	2–3
10. Area	3
11. Surface Area and Volume	3
12. Circles and Other Conic Sections	3

Lessons include the following components:

Exploratory activities appear at the beginning of each textbook chapter (part 1 of each lesson). These activities are a chance to assess what you already know and to play with the ideas in a lesson before the textbook tells you much about them. Complete these activities first, and then check your answers in the appendix of this coursebook.

Exercise sets, found in the textbook, help you develop necessary skills. Please work on them daily, check your answers using the answer key at the back of the textbook or the online solution manual, and correct the problems where you made mistakes. **It is essential that you review and correct any problems you answered incorrectly before moving forward to the next exercise set.** Otherwise, you won't know whether or not you understand the ideas in the lesson. If you are not sure how to correct a mistake, please reach out to your teacher for help.

Chapter tests are found in the textbook at the end of each chapter. After completing a chapter test, you or a supervising adult will grade it and mark the score at the top, such as 18/20. Then, review any mistakes and make necessary corrections. (Students working with a school-based teacher may also be given a different test, which only the teacher will have the answers for.)

Activities are designed to help you apply your learning in new ways and to promote critical and creative thinking. You will be given opportunities to explore real-world applications, dive deeper into concepts with technology, analyze concepts from a historical and cultural perspective, apply math concepts artistically, look at issues in society through a mathematical lens, and more. Most importantly, you get to choose the activity that is most appealing to you. Whether that means investigating a concept you enjoy, challenging yourself with something outside your comfort zone, or exploring your creative side, activities let you take ownership over the direction of your learning!

Share Your Work provides a reminder of what to share with your teacher at the end of each lesson.

It is important that you always show your work and/or explain your thinking, wherever relevant, so your teacher can see where you are having difficulty and better support your learning.

Note: In order to be considered complete, math assignments need to include handwritten computations showing how you arrived at your final answer.

Information About Exercise Sets

- The exercise sets listed are suggestions. More or fewer problems can be done as needed. The textbook includes answers to odd-numbered problems in the exercise sets. You can do even-numbered problems for extra practice, but you will not be able to check your answers.
- If you have access to MyMathLab, you are strongly encouraged to check your work using the online solution manual, which has fully worked out solutions for each problem.
- Primarily, odd-numbered problems are assigned. Note that many assignments suggest completing every other odd (abbreviated “EO odd”), which refers to problems 1, 5, 9, 13, and so on. You may want to circle these problems in the textbook to make sure you are completing the correct ones.
- Concept Extensions can be found at the end of each problem set in the textbook for additional learning and challenges. There are some extension problems included in the assignment list, but you are encouraged to explore as many of these problems as you wish.

- There is a Standardized Test Practice section at the end of each chapter that provides practice for standardized testing. You might consider completing a few of these throughout the course.

Information About Activities and Exploratory Activities

- When submitting your work for each activity, clearly indicate which activity you have completed. Be sure to include all relevant work.
- Many of the activities ask you to answer some reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Any method is acceptable as long as you thoroughly explain your answers and submit them in a format your teacher can access.
- Many of the exploratory exercises and activities in this course are open-ended. This means they do not necessarily have a “right” answer or they have more than one correct solution. There could be multiple ways to approach these problems and multiple ways to answer them correctly.
- In the case of the exploratory exercises at the beginning of each chapter, many are meant to probe your thinking and pique your curiosity about the learning in that lesson. You will not always be able to come up with a definitive answer to those exercises and that is fine! Math is about making observations, asking questions, exploring your curiosity, stretching your thinking, and keeping a flexible and open mind.

Activities will be assessed according to the following criteria.

Problem-Solving and Precision
Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.
Reasoning and Explaining
Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.
Modeling and Using Tools
Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.
Seeing Structure and Generalizing
Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.

Academic Expectations

You are expected to complete your work with integrity and always submit your own original work. The appendix contains important material that you will need to read and incorporate into your work throughout the year.

A Note About the Workload

Students vary greatly in terms of reading speed, reading comprehension, and computational abilities. Some may find the reading in this course takes longer than expected; others may find the math problems or activities take a great deal of time. In general, you can expect to spend about five hours per week on this course. If you need more time to complete the work, you can modify some lessons to focus on fewer assignments or skip activities in some lessons to spend more time on other assignments. Modifications like these will allow you to produce work of a higher quality. Each lesson in this course can be customized to suit your needs.

Keep an eye on the workload as you progress through the course. Make adjustments so you have time for meaningful learning experiences rather than rushing to try to get everything done. Consult with your teacher when making adjustments to the workload.

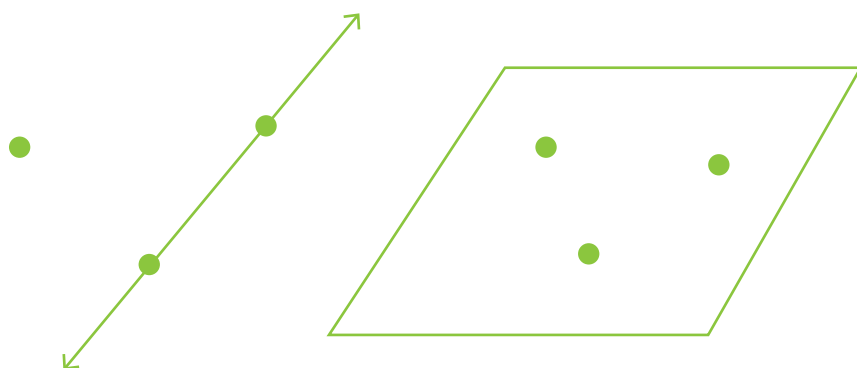
We wish you a challenging and successful year of *Geometry*!

Lesson

1

Part 1: A Beginning of Geometry

Exploratory Activity



Take a minute to analyze the three images above.

Do you know what they are? If you had to write a name and definition for each, what would they be?

Use the GeoGebra applet below to further explore these concepts. (All online resources can be accessed at oakmeadow.com/curriculum-links.)

“Exploring Points, Lines, and Planes”

See the GeoGebra tutorial video in the coursebook introduction if you have not familiarized yourself with GeoGebra yet.

ASSIGNMENT CHECKLIST

- ☐ Complete the exploratory activity.
- ☐ Read the preface and sections 1.1–1.4 and complete the assigned problems.
- ☐ Choose an activity to complete:

Activity A: Integrated Review

Activity B: Which One Doesn't Fit? Basic Terms

Activity C: Error Analysis of Segment Addition Postulate

Lesson Introduction

Suggested time: 1.5 weeks

Geometry comes from the Greek word meaning to “measure the earth,” which is fitting because geometry appears in the world all around us. In this course, we will be exploring Euclidean geometry; it is based on the work of the Greek mathematician Euclid (300 BCE) and begins with three basic terms—**point**, **line**, and **plane**—on which the rest of geometry is based. In this lesson, you will explore these terms and other basic geometric concepts that you will build on throughout this course.

Learning Objectives

Use the checklist below to track how your skills are developing over time, and identify skills that need more work as you progress through part 1 and part 2 of this lesson.

Skills	Notes
Recognize patterns and use them to make predictions	
Understand and apply basic geometric terms and postulates	
Classify types of angles and angle pairs	
Perform basic constructions with a straightedge and compass	
Apply the segment and angle addition postulates	
Find the midpoint and distance between two points	

Exercise Sets

Read the following sections, and then complete the accompanying problem sets. Plan to complete a portion each day. If you have online access to MyMathLab, you can watch the instructional videos as well.

As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read the preface (x) and section 1.1, “Tips for Success in Mathematics” (1).
2. Read section 1.2, “Geometry: A Mathematical System” (11), and then complete the following problems in Exercise Set 1.2.
 - 1–31 odd
 - Extension 33–37 odd
3. Read section 1.3, “Points, Lines, and Planes” (17), and then complete the following problems in Exercise Set 1.3.
 - 1–61 every other (EO) odd
 - Extension 63–79 odd

4. Read section 1.4, “Segments and Their Measure,” (23), and then complete the following problems in Exercise Set 1.4.

- 1–35 odd
- Extension 43 and 45

Activities

Choose one of the following activities to complete.

- Activity A: Integrated Review
- Activity B: Which One Doesn’t Fit? Basic Terms
- Activity C: Error Analysis of Segment Addition Postulate

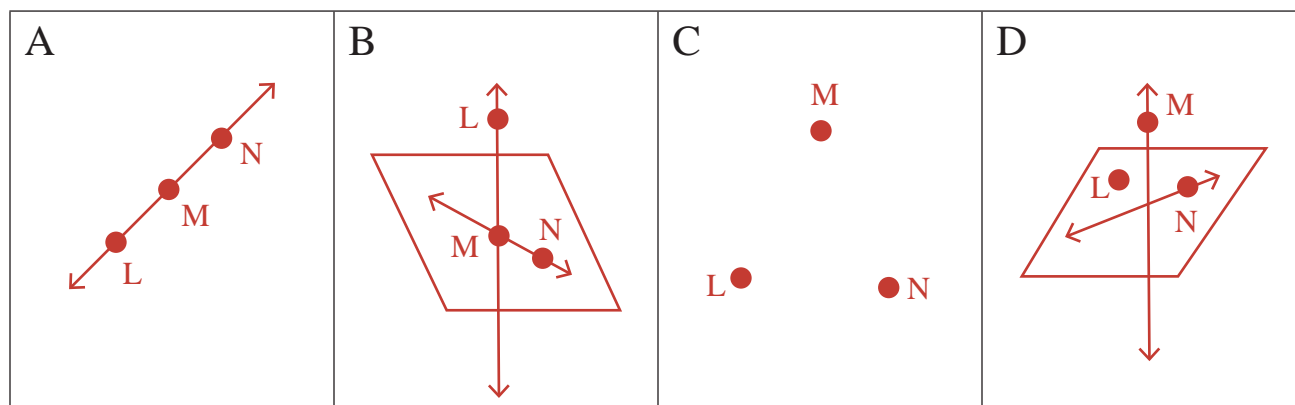
Note: Many of the activities in this course contain reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Regardless of the method, make sure you thoroughly explain your answers. Please consult with your teacher if you have questions about how to submit audio or video recordings.

Activities can be assessed according to the criteria found in the rubric below.

	Notes
Problem-Solving and Precision Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.	
Reasoning and Explaining Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.	
Modeling and Using Tools Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.	
Seeing Structure and Generalizing Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.	

Activity A: Integrated Review

In your textbook, complete the integrated review on page 49, problems 1–14 (all) and 35–45 (all). Show all your work. Check your answers in the textbook appendix, and make corrections to any problems you missed.

Activity B: Which One Doesn't Fit? Basic Terms

Teacher: “Look at the diagram above and answer the question ‘Which one doesn’t fit?’”

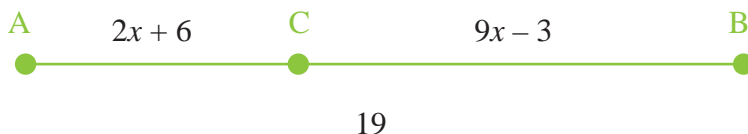
Student: “C doesn’t fit because it is the only one without a line.”

1. How could you further elaborate on the student’s statement about option C?
2. What is another answer to the question “Which one doesn’t fit?” Thoroughly explain your answer.

Activity C: Error Analysis of Segment Addition Postulate

Point C divides line segment AB into unequal parts. $AC = 2x + 6$, $CB = 19$, and $AB = 9x - 3$. Find the value of x , the length of AC, and the length of AB.

A student drew and labeled the following figure and showed the following work to answer this question.



$$\begin{aligned}
 2x + 6 + 9x - 3 &= 19 \\
 11x + 3 &= 19 \\
 11x &= 16 \\
 x &= \frac{16}{11}
 \end{aligned}$$

Examine the student's work. They have not correctly answered the question.

1. What mistake(s) has the student made?
2. What advice would you give to help someone avoid making this/these mistake(s)?
3. Solve the problem correctly.

SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise sets 1.2–1.4 (showing handwritten computations and corrections)
- Choice of activity (labeled with the title of the activity):
 - Activity A: Integrated Review
 - Activity B: Which One Doesn't Fit? Basic Terms
 - Activity C: Error Analysis of Segment Addition Postulate

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.

Lesson

1

Part 2: A Beginning of Geometry

Lesson Introduction

Suggested time: 1.5 weeks

Lesson 1 continues with part 2. Refer to part 1 for learning objectives.

Exercise Sets

Read the following sections, and then complete the accompanying problem sets. Plan to complete a portion each day. If you have online access to MyMathLab, you can watch the instructional videos as well.

As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 1.5, “Angles and Their Measure” (30), and then complete the following problems in Exercise Set 1.5.
 - 1–35 odd
2. Read section 1.6, “Angle Pairs and Their Relationship” (37), and then complete the following problems in Exercise Set 1.6.
 - 1–57 EO odd
 - 59
 - Extension 61 and 63
3. Read section 1.7, “Coordinate Geometry: Midpoint and Distance Formulas” (42), and then complete the following problems in Exercise Set 1.7.
 - 1–29 odd
 - 37–41 odd

ASSIGNMENT CHECKLIST

- ☐ Read sections 1.5–1.8 and complete the assigned problems.
- ☐ Complete the chapter 1 test.
- ☐ Choose an activity to complete:

Activity A: Fill in the Blank—Midpoint and Distance Formulas

Activity B: Explore with Technology Angle Pairs

Activity C: Explore with Technology Constructing Bisectors

4. Read section 1.8, “Constructions: Basic Geometry Constructions” (47), and then complete the following problems in Exercise Set 1.8.
 - 1–25 odd
 - Read page 48
5. Optional: If you would like more practice, you have the option of completing the following, doing as many problems as needed.
 - Chapter 1 Review and Vocabulary Check (49)
 - Chapter 1 Standardized Test Practice (51)

Chapter Test

In your textbook, complete the chapter 1 test on page 50. After completing the test, you or a supervising adult will grade it and mark the score at the top (for instance, 18/20). Then, review any mistakes and make necessary corrections.

Activities

Choose one of the following activities to complete.

- Activity A: Fill in the Blank—Midpoint and Distance Formulas
- Activity B: Explore with Technology Angle Pairs
- Activity C: Explore with Technology Constructing Bisectors

Note: Many of the activities in this course contain reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Regardless of the method, make sure you thoroughly explain your answers. Please consult with your teacher if you have questions about how to submit audio or video recordings.

Activities can be assessed according to the criteria found in the rubric below.

	Notes
Problem-Solving and Precision Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.	
Reasoning and Explaining Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.	

	Notes
Modeling and Using Tools Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.	
Seeing Structure and Generalizing Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.	

Activity A: Fill in the Blank—Midpoint and Distance Formulas

Fill in the x-value and y-value for points A and B using the digits -9 through 9 , no more than once each, for each of the following conditions:

$$A(\square, \square) \quad B(\square, \square)$$

Find two solutions for each problem below. Include all your work justifying your solutions.

1. So that the midpoint of AB is $(2, 5)$
2. So that the midpoint of AB is $(-4, 3.5)$
3. So that A is an endpoint and B is the midpoint with another endpoint $(1, -6)$
4. So that the distance of AB is $\sqrt{34}$
5. So that the distance of AB is $2\sqrt{13}$

Activity B: Explore with Technology Angle Pairs

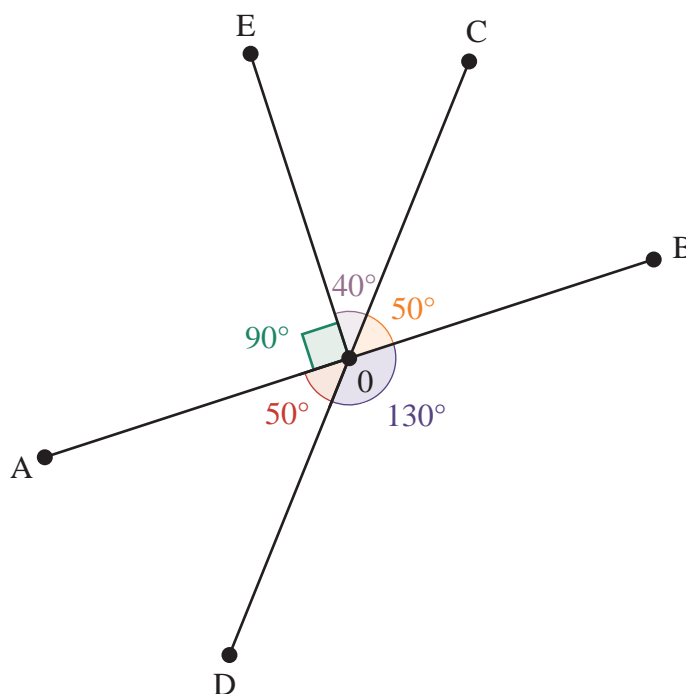
Follow the directions in the GeoGebra file below. (All online resources can be accessed at oakmeadow.com/curriculum-links.)

“Angle Pairs”

See the GeoGebra tutorial video in the coursebook introduction if you have not familiarized yourself with GeoGebra yet.

Answer the following questions. Take a screenshot of the angle orientation you used to answer these questions and include it with your submission.

1. Name a pair of adjacent complementary angles.



2. Name a pair of nonadjacent complementary angles.
3. Name two pairs of supplementary angles.
4. What is the sum of all the angles around point O? Do you think this will always be the case? Why or why not?
5. Name an obtuse angle, an acute angle, and a right angle.
6. Name two pairs of vertical angles.
7. Name two pairs of adjacent angles that are not complementary or supplementary.
8. Name a linear pair.
9. What are the similarities and differences between supplementary angles and a linear pair (in general, not necessarily in this picture)?
10. What is another name for $\angle DOC$?

Activity C: Explore with Technology Constructing Bisectors

Follow the directions in the GeoGebra file below to construct a perpendicular bisector and an angle bisector using technology. (All online resources can be accessed at oakmeadow.com/curriculum-links.)

“Bisector Constructions”

Answer the following questions.

1. How are the methods for constructing a perpendicular bisector of a line segment with a compass and with technology the same? How are they different?
2. How are the methods for bisecting an angle with a compass and with technology the same? How are they different?
3. Which method do you prefer and why?
4. What are some advantages and disadvantages of doing constructions with physical tools versus technology?
5. Include a screenshot of each of your constructions with your submission.

SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise sets 1.5–1.8 (showing handwritten computations and corrections)
- Chapter 1 test
- Choice of activity (labeled with the title of the activity):
 - Activity A: Fill in the Blank—Midpoint and Distance Formulas
 - Activity B: Explore with Technology Angle Pairs
 - Activity C: Explore with Technology Constructing Bisectors

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.

Lesson 2

Part 1: Introduction to Reasoning and Proofs

Exploratory Activity



Can you find the next three images in this pattern?

Lesson Introduction

Suggested time: 1.5 weeks

To find the next images in the sequence above, you are using what is called **inductive reasoning** to infer a pattern and continue it. This is a crucial skill in logical thinking and one you will continue to develop throughout this course. Namely, you will be putting this skill to use when we begin writing **proofs** in this lesson and exploring step-by-step reasoning to support your thinking and arrive at a justified conclusion. This isn't just a skill we use in math either. You practice this every time you justify an argument with reasoning, provide proof to back up a claim, or present logical rationale to your parents about why you deserve the gift you want for your birthday.

Learning Objectives

Use the checklist below to track how your skills are developing over time, and identify skills that need more work as you progress through part 1 and part 2 of this lesson.

ASSIGNMENT CHECKLIST

- ☐ Complete the exploratory activity.
- ☐ Read sections 2.1–2.4 and complete the assigned problems.
- ☐ Choose an activity to complete:

Activity A: Integrated Review

Activity B: Math History—The Discovery of Pi

Activity C: Get Creative with Inductive Reasoning

Activity D: Investigate—Maximizing Area and Density

Skills	Notes
Find the perimeter, circumference, and area of basic shapes	
Identify and extend patterns using inductive reasoning	
Find counterexamples for a given statement	
Identify the hypothesis and conclusion of a conditional statement and write its negation, converse, inverse, and contrapositive statements	
Write biconditional statements and evaluate their truth value	
Determine what makes a good definition	
Identify properties of equality	
Use the Law of Detachment and the Law of Syllogism	
Write a two-column proof using angle theorems and properties of equality	

Exercise Sets

Read and/or watch instructional videos for the following sections, and complete the accompanying problem sets. Plan to complete a portion each day. As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

- Read section 2.1, “Perimeter, Circumference, and Area” (58), and then complete the following problems in Exercise Set 2.1.
 - ☐ 1–39 odd
 - ☐ Extension 41–49 odd
- Read section 2.2, “Patterns and Inductive Reasoning” (65), and then complete the following problems in Exercise Set 2.2.
 - ☐ 1–45 EO odd
 - ☐ Extension 47–53 odd

3. Read section 2.3, “Conditional Statements” (71), and then complete the following problems in Exercise Set 2.3.
 - 1–43 odd
4. Read section 2.4, “Biconditional Statements and Definitions” (76), and then complete the following problems in Exercise Set 2.4.
 - 1–35 odd
 - Extension 47–50 all

Activities

Choose one of the following activities to complete.

- Activity A: Integrated Review
- Activity B: Math History—The Discovery of Pi
- Activity C: Get Creative with Inductive Reasoning
- Activity D: Investigate—Maximizing Area and Density

Note: Many of the activities in this course contain reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Regardless of the method, make sure you thoroughly explain your answers. Please consult with your teacher if you have questions about how to submit audio or video recordings.

Activities can be assessed according to the criteria found in the rubric below.

	Notes
Problem-Solving and Precision Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.	
Reasoning and Explaining Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.	
Modeling and Using Tools Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.	

	Notes
Seeing Structure and Generalizing Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.	

Activity A: Integrated Review

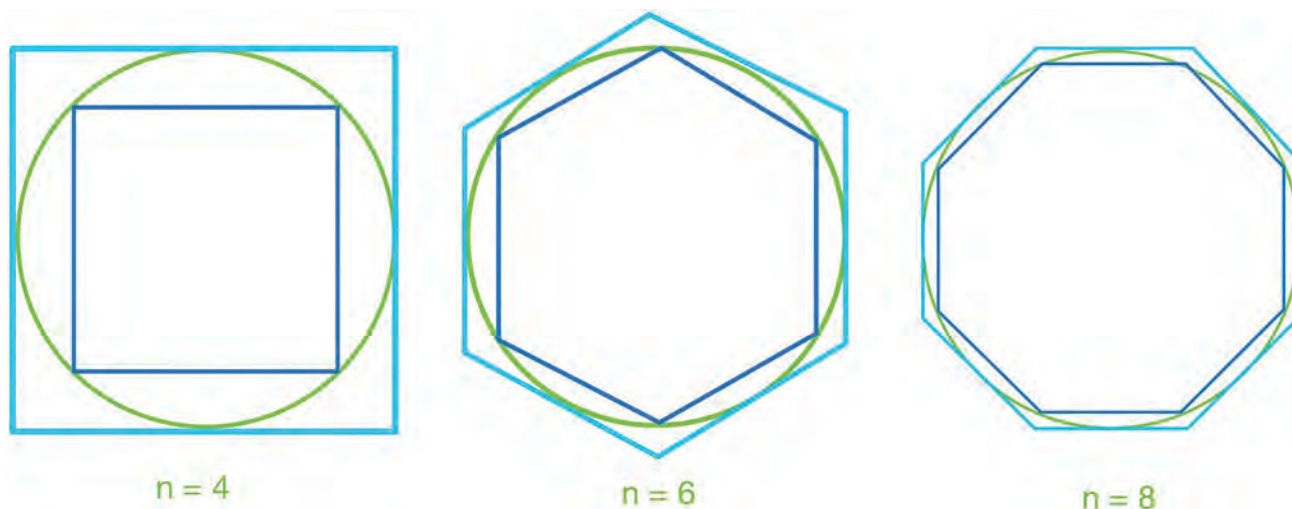
In your textbook, complete the integrated review on page 97, problems 1–23 odd. Show all your work. Check your answers in the textbook appendix, and make corrections to any problems you missed.

Activity B: Math History—The Discovery of Pi

We use the Greek letter π to represent the ratio of the circumference to the diameter of any given circle, meaning if you take the circumference of a circle and divide it by the diameter, the result will be 3.14159 . . . This is a mathematical discovery that was arrived at incrementally over time.

The earliest evidence of pi dates back to the ancient Babylonians and Egyptians around 1900 BCE. There is evidence of the Babylonians using the value of 3 to estimate pi in their area calculations of a circle. They later approximated pi to be equal to $\frac{25}{8}$ (3.125). Pretty good for that time! Likewise, the Egyptians' Rhind Papyrus (1650 BCE) shows an even closer approximation to pi equal to $\frac{256}{81}$ (3.1605). Though these are rough approximations, it is pretty impressive that these ancient civilizations were able to get so close to this very special and intriguing irrational number.

Archimedes of Syracuse (287–212 BCE) was one of the greatest mathematicians of his time. He began exploring pi and developed a method for approximating its value by trying to measure the circumference of a circle with a diameter of 1. (If we think about this knowing what we know today, the circumference of a circle is $C = \pi d$, so if $d = 1$ then $C = \pi$ and calculating the circumference of the circle would equate to calculating pi.) He did this by inscribing and circumscribing polygons around a circle to approximate the circumference. As he increased the number of sides of the polygons, he got closer to the actual value of the circumference. For example, you can see in each image below that there is an inscribed polygon (dark blue) whose perimeter is less than the green circle and a circumscribed polygon (light blue) whose perimeter is more than the green circle. This provides a lower and upper bound in between which the circumference must lie. You can see as the number of sides of the polygon increases, the closer the lower and upper bounds get to the actual circumference. Archimedes repeated this process until he reached a polygon with 96 sides! This gave him a range of $3\frac{10}{71}$ and $3\frac{1}{7}$ for the circumference, which means he knew that pi was somewhere between 3.1408450704 and 3.1428571429.



Later, a Chinese mathematician and astronomer named Zu Chongzhi (429–501 CE) got even closer. Though he was likely unfamiliar with Archimedes’s work, he used a similar method with an inscribed polygon with 24,567 sides! He approximated pi to be the ratio of the circumference to the diameter equal to $\frac{355}{113}$ (3.14159292).

In the fifteenth century, an Indian mathematician named Madhavan of Sangamagrama accurately calculated pi to 11 decimals. In 1707, a Welsh mathematician named William Jones first assigned the Greek letter, π , from the Greek word for “perimeter” to represent this ratio. The symbol was popularized by Leonard Euler 30 years later. The most accurate calculation of pi before computers was done by mathematician D. F. Ferguson, who correctly calculated 808 digits of pi in 1947.

Now, it is your turn to calculate pi!

Materials

- 5 or more circular objects
- measuring tape with centimeters (like those used for sewing), or string/twine/ribbon and a ruler with centimeters
- calculator
- chalk, tape, or pencil

Part 1

Fill in the following table by measuring the diameter and circumference of your five circular objects. For the circumference, use the tape measure or string to mark the distance around the object and then lay it flat on the ruler to figure out the measure. Complete your measurements in centimeters.

Circular Object	Diameter (in cm)	Circumference (in cm)	Circumference ÷ Diameter

1. What did you notice about the circumference ÷ diameter?
2. Were you able to calculate pi exactly? What factors might have contributed to variations in your calculations?

Part 2

A practical application of circumference is figuring out the distance wheels of various sizes can travel. Explore the relationship between the distance traveled on the ground and wheel size—you can do this with an actual wheel, like on a bicycle, or any other round object that you can roll and track.

Measure the distance that at least two wheels of a different size travel.

- Mark a spot on the wheel (with chalk, tape, pencil, or some other nonpermanent means).
- Place that spot on the ground and mark the location with chalk or tape.
- Roll the wheel one complete rotation until the spot is on the ground again and mark the ground.
- Measure the distance from start to finish.
- Repeat for another circular object of a different size.
- Fill in the following chart.

Circular Object	Diameter (in cm)	Circumference (in cm)	Distance Traveled

1. What do you notice?
2. How many rotations would your smaller wheel need to complete to travel the same distance as the larger wheel?
3. What generalizations can you draw about wheel size, rotations, and distance?

Activity C: Get Creative with Inductive Reasoning

Create your own inductive reasoning pattern like you studied in section 2.2. You can make it with shapes, numbers, or both. Make it as creative as you would like, but it should be appropriately challenging.

1. Have at least two people see if they can find the next three figures/numbers in the pattern you created. Verify their answers.
2. How did your participants do with the challenge? Did they provide you with any insights or feedback that you found useful?
3. What was your favorite part about your creation? Why?
4. If you were to do this again, is there anything you would do differently? Why or why not?
5. Submit your pattern and your participants' answers along with your responses to the questions above.

Activity D: Investigate—Maximizing Area and Density

Imagine an animal sanctuary is building an enclosure for their meerkat population. They want to ensure that they create a habitat that has the largest area possible using the 220 feet of fencing they have. The only stipulation the sanctuary has is that the enclosure cannot be a triangle.

Using what you know about the perimeter and area of the shapes you studied in this chapter, determine the shape and dimensions of the enclosure that will provide the meerkats with the most possible space.

1. What is the shape and dimensions of the enclosure you would propose to the animal sanctuary?
2. What strategies did you use to find your answer? Include all work with your submission.
3. The sanctuary has determined that a single meerkat needs at least 300 square feet to live a happy life. What would be your recommendation on how many meerkats can live in your proposed enclosure? Justify your answer.

SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise sets 2.1–2.4 (showing handwritten computations and corrections)
- Choice of activity (labeled with the title of the activity):
 - Activity A: Integrated Review
 - Activity B: Math History—The Discovery of Pi
 - Activity C: Get Creative with Inductive Reasoning
 - Activity D: Investigate—Maximizing Area and Density

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.

Lesson 2

Part 2: Introduction to Reasoning and Proofs

Lesson Introduction

Suggested time: 1.5 weeks

Lesson 2 continues with part 2. Refer to part 1 for learning objectives.

Exercise Sets

Read and/or watch instructional videos for the following sections, and complete the accompanying problem sets. Plan to complete a portion each day. As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 2.5, “Deductive Reasoning” (82), and then complete the following problems in Exercise Set 2.5.
 - 1–31 odd
2. Read section 2.6, “Reviewing Properties of Equality and Writing Two-Column Proofs” (88), and then complete the following problems in Exercise Set 2.6.
 - 1, 5, 9
 - 11–27 odd
 - Extension 29–33 odd
3. Read section 2.7, “Proving Theorems About Angles” (94), and then complete the following problems in Exercise Set 2.7.
 - 1–37 odd
 - Review and Preview 47–51 odd
4. Optional: If you would like more practice, you have the option of completing the following, doing as many problems as needed.

ASSIGNMENT CHECKLIST

- Read sections 2.5–2.7 and complete the assigned problems.
- Complete the chapter 2 test.
- Complete Activity A: UNO Proofs.
- Optional:
 - Activity B: Which One Doesn't Fit? Deductive Reasoning
 - Activity C: Fill in the Blank—Angle Theorems

- Chapter 2 Review and Vocabulary Check (97)
- Chapter 2 Standardized Test Practice (99)

Chapter Test

In your textbook, complete the chapter 2 test on page 98. After completing the test, you or a supervising adult will grade it and mark the score at the top (for instance, 18/20). Then, review any mistakes and make necessary corrections.

Activities

Complete Activity A: UNO Proofs.

Optional:

- Activity B: Which One Doesn't Fit? Deductive Reasoning
- Activity C: Fill in the Blank—Angle Theorems

Note: Many of the activities in this course contain reflection questions. You may choose to answer these questions in writing, as an audio recording, or as a video recording. Regardless of the method, make sure you thoroughly explain your answers. Please consult with your teacher if you have questions about how to submit audio or video recordings.

Activities can be assessed according to the criteria found in the rubric below.

	Notes
Problem-Solving and Precision Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used.	
Reasoning and Explaining Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning.	
Modeling and Using Tools Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution.	

	Notes
Seeing Structure and Generalizing Structures and patterns are identified and extended to make generalizations and/or connections to prior learning.	

Activity A: UNO Proofs

Proofs will be explored throughout this course and, therefore, it is essential to understand the basic structure, purpose, and flow of proofs. This activity will help acquaint you with the fundamentals of proof writing using UNO cards and help you build skills that will aid you throughout the remainder of the course.

If you have a deck of UNO cards, feel free to use them in this activity, but it is not necessary to have a deck to complete this task. You just need to have a basic understanding of the rules of the game and card meanings. (All online resources can be accessed at oakmeadow.com/curriculum-links.)

1. Familiarize yourself with UNO if you don't know the game. Use the following links to watch and/or read how to play the game.

“How to Play UNO”

“UNO (Game): The Rules & How to Play—According to Mattel”

2. Go through the following lesson to learn about UNO proofs.

“Proofs with Uno”

3. Complete this worksheet and try UNO proofs on your own.

“Proofs with Uno Assignment”

Note: The last exercise asks you to apply the skills you have learned to a geometric proof. To complete this proof, you will need to reference a theorem that we haven't quite gotten to yet: three angles inside a triangle add up to 180 degrees.

4. What was your favorite part of this activity? Explain why and submit your response with your proofs.
5. Optional: Find a partner and enjoy an actual game of UNO!

Activity B: Which One Doesn't Fit? Deductive Reasoning

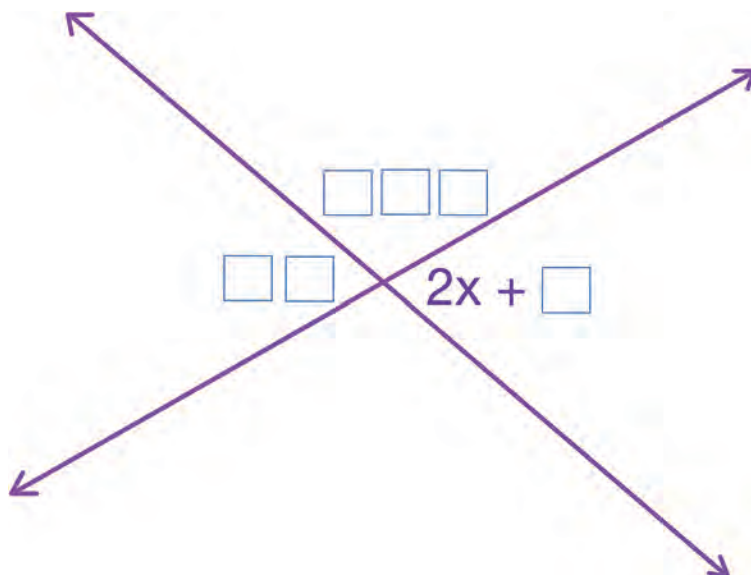
<p>A</p> $p \rightarrow r$ $r \rightarrow p$	<p>B</p> $p \rightarrow q$ $q \rightarrow r$	<p>C</p> $p \rightarrow q$ <p>p is true</p>	<p>D</p> $p \rightarrow \sim q$ $\sim p \rightarrow q$
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Which one doesn't fit?

Assume the given statements are true. Explain your answer and any conclusions you can draw from the statement. Then, create an example to illustrate your reasoning and conclusion.

Activity C: Fill in the Blank—Angle Theorems

Use the digits 1 through 9, no more than once each, to fill in the blue squares so that x is a whole number. Come up with two different solutions and show all your work for solving the problem.

**SHARE YOUR WORK**

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise sets 2.5–2.7 (showing handwritten computations and corrections)
- Chapter 2 test
- Activity A: UNO Proofs

- Optional choice of activity (labeled with the title of the activity):
 - Activity B: Which One Doesn't Fit? Deductive Reasoning
 - Activity C: Fill in the Blank—Angle Theorems

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.



Appendix

Bonus Unit	154
Answer Key to Exploratory Activities	155
Answer Key to Even-Numbered Problems in Exercise Sets.....	156
Academic Expectations	158
Works Cited	159