Advanced Mathematics

An Incremental Development

Second Edition

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SAXON PUBLISHERS, INC.

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Advanced Math Syllabus

Textbook

Advanced Math: An Incremental Development (Saxon)

Materials

Advanced Math textbook
Homeschool Packet: Form A Tests, Answers to Problem Sets and Form A Tests

Schedule of Assignments

As an independent study student, you can complete this course at your own pace. Some students benefit from doing every single problem in the problem sets while others prefer to do all the even numbered problems, and then go back and do the odd numbered problems for skills that need extra practice. By checking your answers after each problem set, you can gauge how well you are grasping the material and adjust your work load accordingly.

There is a proposed schedule of assignments on the back that will help you move through the course in 31 weeks. Feel free to adjust this schedule to suit your needs and learning style.

We wish you a challenging and successful year of Advanced Math!

Advanced Math Assignment Sheet

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