Math Connections

Teacher Edition

Oak Meadow

Oak Meadow, Inc.
Post Office Box 615
Putney, Vermont 05346
oakmeadow.com

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Teacher Edition Introduction

This course is designed to give students opportunities for real-world applications of mathematics. It is intended for students who have already taken Algebra I and Geometry courses. Students will engage in mental math, math journaling, and a wide variety of activities, as well as prepare two large projects.

The textbook, *Math for Your World* (Pearson 2016), is an excellent resource and includes clear explanations of the many concepts and skills covered this year. The answer key at the back of the book includes answers to all odd-numbered Practice and Application Exercises. For this reason, students are assigned these exercises so that they can check their answers and, if mistakes are made, go back and figure out why. In this way, students can direct their own learning and get immediate feedback on their work. Students are also encouraged to choose (with the help of an adult, if necessary) how many practice problems are needed in order for them to become comfortable working with a particular skill or concept.

Students are always required to show their steps when completing a problem. Simply writing the answer is not enough (especially since the answers can be found in the back of the textbook). Showing each step helps the student make sure to proceed logically through a problem, and makes it easy to go back and detect where an error occurred, if necessary.

In this teacher edition, you will find the text for all assignments and some activities, and answers for those activities that pose a specific question. Teacher edition answers are shown in **orange**. You will also find links to many excellent online resources on the *Math Connections* resource page on the Oak Meadow website at www.oakmeadow.com/curriculum-links.

If you are homeschooling independently, this teacher edition can serve as your support as you guide and evaluate your student's work. When a student gets a factual answer wrong, you can share the correct answer and address any underlying misconceptions. The focus should always be on the learning process rather than on a sense of judgment. Several incorrect answers related to a particular topic point to an area the student will benefit from revisiting.

For obvious reasons, it is best not to share this teacher edition with your student. Each student is expected to produce original work, and any incidence of plagiarism should be taken very seriously. If you notice a student's answers matching those of the teacher edition word for word, a discussion about plagiarism and the importance of doing original work is necessary. While students in high school are expected to be well aware of academic honesty, any discussion about it should be approached as a learning opportunity. Make sure your student is familiar with when and how to properly attribute sources.

We encourage you and your student to explore the topics introduced this year through dynamic exchanges of ideas, partnering on mental math activities, viewing and discussing the many video resources mentioned in this course, and in other active, experiential ways. We hope this course leads your student into a better understanding of mathematics and its importance in the wider world.

Coursebook Introduction

Welcome to *Math Connections*! This course will answer the common math question, “When will I ever use this?” The overarching theme of this course is problem-solving in the real world. The first chapter introduces problem-solving techniques that set the stage for the entire course. It is not always easy to solve problems and it is important that you do not give up when you don’t immediately understand a problem or see how it will be solved. This course is designed to help you strengthen both your math skills and your ability to persevere through challenges.

You will be asked to explore historical topics to illustrate the challenges and processes that mathematicians—people just like you—worked through to arrive at the discoveries that we take for granted today. This will, hopefully, help you appreciate the nature of problem-solving as a challenging and rewarding process even when a solution is difficult to achieve. As the famed inventor Thomas Edison is credited with saying, “Results? Why, man, I have gotten lots of results! If I find 10,000 ways something won’t work, I haven’t failed. I am not discouraged, because every wrong attempt discarded is often a step forward.” When we think about “wrong attempts” as being a necessary step on the path to success, it empowers us to get back up and keep moving.

Some of the material in the textbook chapters will likely look familiar to you from Algebra I and Geometry, but this overlap is intentional. In *Math Connections*, we will review the basics so you have the opportunity to master concepts you may have missed in an earlier course and solidify your understanding of the underlying mathematical properties and structure. We will then dig more deeply, applying topics to real-world problems.

Other topics will probably be new to you, and may include ideas that you wouldn’t typically think of as being “math.” In this course, we will explore connections to other areas of math, as well as to various disciplines such as history, literature, science, art, and philosophy, in order to introduce you to the beauty and wonder that is mathematics. Try to approach unfamiliar topics with an open mind and a sense of curiosity.

How the Course Is Set Up

This two-semester course consists of 13 lessons, plus a midterm project and a final project. The first semester contains lessons 1 through 6 and the midterm project. The second semester contains lessons 7 through 13 and the final project. Each lesson corresponds to one of the 13 chapters in the textbook, Blitzer’s *Math for Your World* (Pearson 2016).

Each lesson provides an estimate of how long it may take to complete, ranging from one to three weeks per lesson. Please note, however, that some lessons may take more or less time, depending on your experiences and abilities. The time estimate is a general guideline, so adjust your own pace as it fits for you.

In each lesson, you'll find the following types of assignments:

Mental Math

Mental math is any calculation that you do in your head. Mental math skills are important for completing tasks and making decisions in everyday life. Mental math also involves *how* to think mathematically to solve problems and understanding *why* certain steps are used. Throughout this course, we will do mental math as a warm-up activity for each day you work on math. Within each lesson, there will be a new mental math technique, strategy, or game for you to add to your "bag of tricks"—your tool kit of mental math skills. The more mental math practice you do, the better you will get at making mental calculations and the better you will understand the *how* and *why* of what you do. The key is to get into the habit of regularly doing the mental math activities. As with learning music, it is better to practice a single short exercise per day than to do them all in a longer session once per week. The mental math activities are designed to take only a few minutes each day.

Readings and Exercise Sets

Each textbook chapter is divided into sections that include Checkpoints and an Exercise Set. Read one section at a time, completing the Checkpoint problems along the way. At the end of each section, complete the Exercise Set before moving on to the next section. You will find answers in the back of the textbook—this lets you check your work and make corrections right away. Most Exercise Sets will include the following:

- **Concept and Vocabulary Exercises** can be done verbally with your home teacher.
- **Practice Exercises** provide a wide variety of problems. You will choose a selection of odd-numbered problems from each section to complete. (The answer key only includes answers to odd-numbered Practice Exercises, so it's important to make your selections from odd-numbered problems only.) This lets you customize the practice work to your own needs. You may only need to practice 10 problems in one section, yet need to practice 20 or more problems in a section that you find more challenging. It is very important to make sure that you choose enough problems to ensure that you fully understand the concepts, and that you cover at least one of every kind of problem. It is your responsibility (with the help of your home teacher) to determine the problems you should complete.
- **Application Exercises** help you develop your problem-solving skills and see how the concepts in the chapter connect with real-world problems. In most lessons, you will be completing all odd-numbered Application Exercises.
- **Critical Thinking and Technology Exercises**, found at the end of some Exercise Sets, are optional and can be done verbally or in written form. You do not have to do the Group Exercises.

Math Journaling

Have you ever easily solved a problem but drawn a blank when you were asked to explain *how* you solved it? Being able to understand what you are doing in math and to clearly communicate your ideas are valuable and essential components of learning and doing mathematics. Each lesson in this course contains one or more journal assignments. Writing journal entries will help you develop critical thinking skills, synthesize and apply the concepts you have learned, reinforce your math vocabulary, and strengthen your ability to communicate your mathematical thinking.

Activities

At the end of each lesson, you will tie together everything you have learned in the lesson with an activity. Activities are deeper application problems that allow you to explore the lesson's concepts in a concrete or thought-provoking way. Some activities will have you connect what you learned to another discipline, such as art, history, English, or philosophy. Other activities will explore the work of a mathematician, make use of online tools and games, and require you to use research and problem-solving skills to gather information and make an informed decision.

Tests

Tests in this course are designed to be open book. You may refer to the textbook and your notes as needed. The Chapter Test at the end of each textbook chapter serves as the test for students who are completing the course independently. For students enrolled in Oak Meadow School, it serves as an optional—but recommended—practice test. Enrolled students are required to complete an alternate test that will be supplied by the school.

Midterm and Final Projects

Each semester contains a large-scale project due in the final week of the semester. The midterm project assignment is to write a creative biography on a mathematician of your choice. The final project assignment is to research and create a project on a topic of your choice that relates to math. Each project will be worked on in stages throughout the semester. Certain lessons contain Project Milestone assignments that will guide you through the project development process and keep you on track to complete the project by the end of the semester.

People in Mathematics

Each lesson contains a short biographical sketch of a mathematician whose work relates in some way to the chapter's topic. These individuals were selected because of their significant contributions to mathematics and, in some cases, because of their strong character in the face of adversity. They can serve as an inspiration to everyone facing challenges, reminding us to persevere and solve problems.

Appendix

At the end of this coursebook, you will find an appendix that includes original work guidelines, information on how to avoid plagiarism, and detailed instructions on finding reputable sources and citing sources correctly. Take a few minutes to look over this information so you can refer back to it as needed throughout the year. You will be expected to know this information and use it in your work.

Course Expectations and Tips

Here are some guidelines to help you get the most out of this course.

- Carefully read the information in the coursebook and textbook. The textbook was selected for its excellent readability and step-by-step example solutions. It's worth taking the time to thoroughly read the textbook assignments and follow along with the examples.
- Do not automatically skip material that looks familiar. Skipping over review topics may seem like a way to save time and energy, but mathematics is a subject that builds on itself, so it is necessary to periodically review concepts in order to keep them fresh and provide a solid foundation from which to learn the next layer of material. Taking the time to read through examples and work through problems, even those that contain ideas that you are already familiar with, will help you to better understand the underlying mathematical structure and ready your mind for new ideas.
- Do the Checkpoint problems after each example, and compare your answers to the ones in the back of the book. This will help you gauge whether you understood the problem. Tip: Bookmark the solutions page you are on with a sticky note for quick and easy reference.
- Always check your Exercise Set solutions with the answers in the back of the textbook and make corrections as appropriate. This is a very important step in the independent learning process as it gives you an opportunity to assess your understanding of the course material as you learn it. Ask for help if you get stuck!
- Show all of your steps for test problems. Any step that cannot easily be done mentally must be written down in an organized and mathematically valid format. If a problem does not require multiple steps, then it is wise to write a short explanation indicating how you arrived at your answer. In order to get into the habit of showing your steps on tests, you should practice showing your steps on your Practice Exercises, as well.
- Use tools such as a ruler, compass, and protractor as appropriate to make your graphs and diagrams neat and precise.
- Always check your answers for completeness and label answers with units.
- Do the mental math warm-up each day. Daily practice keeps ideas fresh in your mind, trains you to work with new ideas, and over time you will notice that you can make calculations faster and your overall math performance will be stronger. It's important to be consistent about your mental math practice. Don't skip these exercises!

- Write thoughtful, well-developed journal responses. You may choose to write journal responses by hand or type them out. While perfection is not expected, your responses should be clearly written and make sense. Check over your writing and make quick edits as necessary to ensure readability.

Use of Technology

A calculator will be needed for this course. If you plan to eventually take Algebra II, Advanced Math, or Calculus, a graphing calculator will be required for those courses. You are encouraged to invest in a graphing calculator (such as the TI-83 Plus or TI-84 Plus) early on so you can become familiar with its use. These calculators are frequently used in college math and science courses as well, so you should get a lot of use out of this tool.

If you do not plan to take the courses listed above, then a scientific calculator will suffice. Please note: A simple four-function calculator will *not* be powerful enough for use in this course. A scientific calculator has buttons for trigonometric functions (sin, cos, and tan) and also has a key for exponents.

You are permitted to use your calculator on all assignments, with the exception of mental math exercises that are intended to be done mentally.

This course also makes use of internet resources including websites, videos, and online games. You will need internet access and a device with a browser capable of viewing and running these sites and applications. The course can be done without using the internet, but accessing these online sources can enhance your learning experience. Enrolled students who do not have access to the internet should contact their teacher to make alternate arrangements.

For Students Enrolled in Oak Meadow School

At the end of each lesson, you will see a section **Share Your Work** where you will be reminded of what to submit to your teacher. Here are some additional notes:

- The journal entries and activities are submitted to your teacher at the end of each lesson and will be included in your grade for the lesson.
- Exercise sets are not submitted to your teacher and will not be included in your grade for the lesson. However, working on exercise sets is a necessary step in the learning process.
- When you submit a Project Milestone assignment to your teacher, it is necessary to get approval before moving on to the next project milestone.
- **Always show your work.** Simply writing down the answer is not enough. You will be marked down if you do not show your work.

You will find a Lesson Test for each chapter in the testing packet that you download from the Oak Meadow Gateway. The Lesson Test is submitted to your teacher and will be included in your grade for the lesson. Take the Lesson Test only after you have fully reviewed and prepared for it. We recommend that you first take the textbook's Chapter Test and use it as a practice test to identify any lingering trouble spots. If you are still confused when making corrections, it is important that you ask for help from your home teacher or teacher. It is always best to resolve trouble spots *before* taking the graded test.

Let the Journey Begin

Now that you have a good idea of what to expect, get ready to experience math in a new way. We hope you'll come to appreciate math and see it as an important tool in your life.

Lesson 1

Problem-Solving and Critical Thinking

We will begin this course by exploring techniques for solving and thinking critically about problems—not just mathematical problems, but any type of problem we could encounter in our everyday lives. Then, using a four-step problem-solving process, you will practice looking for patterns and solving a variety of math problems and puzzles.

This lesson should take approximately two weeks to complete.

Learning Objectives

- Distinguish between and use inductive and deductive reasoning.
- Use estimation techniques to find approximate answers to problems.
- Explain the purpose and features of circle graphs, bar graphs, and line graphs.
- Apply estimation techniques to information presented on graphs.
- Estimate relationships between variables through mathematical modeling.
- Solve problems using the four-step problem-solving process.
- Explore puzzles and games that involve problem-solving.

Why It Matters

While numbers are often the first thing that comes to mind when talking about math, the truth is that mathematics is also the study of several other ideas, including patterns. Patterns play a huge role not only in mathematics but in all disciplines, from the identification

ASSIGNMENT CHECKLIST

- ☐ Mental Math Set A: The Sums Game
- ☐ Mental Math Set B: Rounding and Estimation Games
- ☐ Read Chapter 1 in textbook.
- ☐ Complete a selection of exercises for sections 1.1 through 1.3.
- ☐ Read Chapter 1 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: Math and You
- ☐ Math Journal B: The Four-Step Problem-Solving Method in Action
- ☐ Activity A: Modeling College Graduation Rate
- ☐ Activity B: Figurate Numbers and Pascal's Triangle
- ☐ Activity C: Logic Puzzles

> *"A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on mind and character for a lifetime."*
>
> **George Pólya, *How to Solve It***

of themes in a novel or the exploration of motifs in a work of art or music to recognizing the similarity of repeated events in history or observing trends in the impact humans have on the environment. Patterns are everywhere, but we need to recognize them and clearly articulate their structure in order to show their significance.

Problem-solving is another aspect of mathematics that connects to many other areas of life. There are always problems to be solved, but doing so can be quite a challenge! Fortunately, there are tools that can help us with the problem-solving process. By practicing with these tools, we can train our brains to look for patterns and generate ideas for solutions. We can then apply these skills to solve problems in all areas of our lives and to help make the world a better place.

Mental Math Warm-Ups

Each lesson will contain at least one set of mental math activities. These exercises are meant to strengthen your mental math skills over time and to help you learn and discover new techniques to use in your everyday life. They are intended to be quick, so spend just three to five minutes practicing at the beginning of each math session.

You are encouraged to do your mental math exercises with a partner to enhance learning and make the process more fun. Ask a parent, tutor, sibling, or friend to be your partner. You can make it a collaborative effort where you make up and solve problems together, or you could create some friendly competition by challenging your partner to beat the clock or see who gets the most correct answers. Feel free to adapt the mental math exercises in any way that will make them more fun and engaging for you.

If you are unable to work with a partner, the mental math exercises can be adapted for solo practice by creating a short list of problems to challenge yourself with. You should write your problem list on a piece of paper. As you work, write down your answers, but **all calculations should be done in your head.** You can then use a calculator to verify your answers. To make things more interesting, you could challenge yourself to beat the clock or to get all answers correct.

Note for students enrolled in Oak Meadow School: Mental math activities are not submitted to your teacher. They will not appear on tests, and do not count toward your grade. However, it is expected that you will complete the mental math activities as they will strengthen your overall mathematical skills and understanding.

This lesson contains two sets of mental math warm-ups:

- Mental Math Set A: The Sums Game
- Mental Math Set B: Rounding and Estimation Games

Complete one set each week.

You are encouraged to partner with your student for these activities. This will show your interest in what your student is learning, allow you to determine areas in which your student is confident and areas that need more work, and let you share in the enjoyment of the challenges and successes of each activity.

Mental Math Set A: The Sums Game

For the first week of mental math, you will be strengthening your mental addition, subtraction, and grouping skills through a fast-paced game. It is best to work with a partner, but you could also play the game on your own. Spend two to three minutes playing this warm-up game at the start of each math session to sharpen your mental math skills. Even better, play it anytime you're bored (riding in the car or in line at the store, for example).

Day 1: The goal of the first Sums Game is to find pairs of numbers that add up to 100. One person calls out a number between 0 and 100 and the other person must mentally figure out what number must be added to the first number to get to 100. For example, if your partner gives the number 64, you would answer 36 because 64 + 36 = 100. Take turns calling out numbers for each other. As you get the hang of the game, try to answer more and more quickly. Afterward, discuss with your partner what strategies you both used to find the numbers. Did you use the same strategies? Perhaps your partner found a trick that you can try next time.

Days 2 and on: Continue to play the Sums Game, but choose a different target number each day, perhaps 200, 500, or 1,000. Gradually make the game more challenging by including negative numbers, fractions, and decimals. For example, you could make your target number 1 and find pairs such as .35 and .65, or $\frac{1}{4}$ and $\frac{3}{4}$. Continue to discuss strategies with your partner.

Mental Math Set B: Rounding and Estimation Games

For your second week of mental math, you will strengthen your mental rounding and estimation skills through some friendly, fast-paced competition.

Day 1: With a partner, take turns giving each other a number, along with a place value to round to. For example, you might say "780. Round to the nearest hundred." Your partner would answer with "800." Gradually make the problems harder by including decimals and fractions, such as $32\frac{2}{3}$ rounded to the nearest tenth. (Answer: 32.7.) Try to think faster as you go, increasing the pace of your game. If you do not have a partner, jot down several numbers and place values to round to, then race the clock to see how many you can answer in three minutes.

Day 2: Either play this game in a store or refer to a grocery store sale flyer. You can play it alone or with a friend. Make a grocery list (real or imagined) with several items. Also list the price of each item. Mentally round and sum the prices to get an estimate of the total cost of the items. (Bonus points if you have coupons that you can mentally deduct from the total and use to save money!) Then compare the estimate with the actual value, which you or your friend can find using a calculator. If playing against a friend, see which of you can come closer to the actual price.

Days 3 and on: Repeat the previous activities, making the problems more and more challenging, and the pace quicker and quicker. Try making calculations involving multiplication and division too. Can you feel your mental math muscles getting stronger?

Assignments

Textbook Assignments and Test

1. Read textbook sections 1.1 through 1.3 in *Math for Your World* (Blitzer 2016). For each section, follow along with the examples and try the Checkpoint problems. Check your answers against those at the back of the textbook. Verbally answer the Concept and Vocabulary Check exercises at the end of the section. Check your answers with those at the back of the book.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 1.1 through 1.3 (choose from the odd-numbered problems only). Choose several problems of each type to ensure sufficient practice.
3. Do all odd-numbered Application Exercises for Exercise Set 1.2 and 1.3 (no Application Exercises are necessary for Exercise Set 1.1). Check your answers with the back of the book. Make any necessary corrections and review areas that need work. If you need additional practice, you may want to complete a selection of even-numbered problems.
4. Review the Chapter 1 Summary at the end of the chapter. If you feel you need additional practice in any area, select problems from the Chapter 1 Review.
5. Complete the Chapter 1 Test from the textbook (for independent students) or the Lesson 1 Test from the test packet (for enrolled students). Students who are using the curriculum independently will complete the test in the textbook and check their answers in the back of the book. Make necessary corrections and review areas that need work. Students who are enrolled in Oak Meadow School must complete the Lesson 1 Test from the test packet and submit it to their teacher for grading.

Reminder for enrolled students: You are encouraged to use the Chapter 1 Test in the textbook as a practice test. This will allow you to check the answers in the back of the book to ensure that you have mastered all major concepts in the chapter. If there are any areas that need work, review and practice as needed before taking the Lesson 1 Test from the test packet.

Math Journal

Complete both journal assignments (do one per week):

- Journal A: Math and You
- Journal B: The Four-Step Problem-Solving Method in Action

Ask your student to share the math journal at the end of each lesson. This journal is primarily a learning tool for your student, but it can also give you insight into your student's understanding of each topic. When a specific response is required, the journal activity will be included here.

> *"If there is a problem you can't solve, then there is an easier problem you can solve: find it."*
>
> **George Pólya, *How to Solve It***

Journal A: Math and You

This assignment will help you explore your relationship with mathematics while introducing you to math journaling.

Consider the following questions: When you hear the word "math," what comes to mind? What experiences have you had in learning math? In what ways do you use math outside of math class? If you could study or do anything involving math, what would it be? If you could meet any mathematician, past or present, who would it be?

Choose one or more of these questions and begin to write. Don't worry about the mechanics of your writing; just let your ideas flow for five to ten minutes. When you finish, read your response and quietly reflect on it for a moment. Have you ever really thought about these questions before? Do your answers surprise you?

This purpose of this journal assignment is to get students thinking about how their past experiences with math have shaped their current attitude toward the subject. This can often prove enlightening for students, enabling them to recognize what they like or dislike about math, and understand why they have developed those attitudes. It can also provide insight into how they best learn math and what has and hasn't worked for them in the past. This journal entry may bring up some past emotional experiences for the student, so there may be some uncomfortable feelings about sharing it. If your student does share it with you, offer encouragement and remind your student about past accomplishments in math—especially times when hard work and perseverance paid off. This journal entry could be a good starting point for a conversation about goals and strategies for this course, but try to keep such discussion light, without criticism or pressure.

Journal B: The Four-Step Problem-Solving Method in Action

A great feature of the four-step problem-solving method is that it's useful for solving more than just math problems! In this assignment, you will apply the method to solving a problem in your own life.

For this journal entry, do the following:

1. Give an example of an everyday problem you might encounter.

2. Devise a plan for solving the problem using Pólya's four-step problem-solving process. (Review section 1.3 in the textbook if necessary.)
3. Explain how you might carry out your plan. Be sure to list each of Pólya's four steps and briefly describe how it applies to your problem.

   If you're stuck for an idea for a problem, try some of these: how would you go about organizing a notebook, planning out your schedule, or determining the fastest driving route to a friend's house? These are just a few suggestions; you are encouraged to come up with your own ideas.

In assessing this assignment, look for a clearly defined problem statement and an appropriate response for each of the four steps of the problem-solving method described in the textbook.

Activities

Complete all three activities below.

- Activity A: Modeling College Graduation Rate
- Activity B: Figurate Numbers and Pascal's Triangle
- Activity C: Logic Puzzles

Activity A: Modeling College Graduation Rate

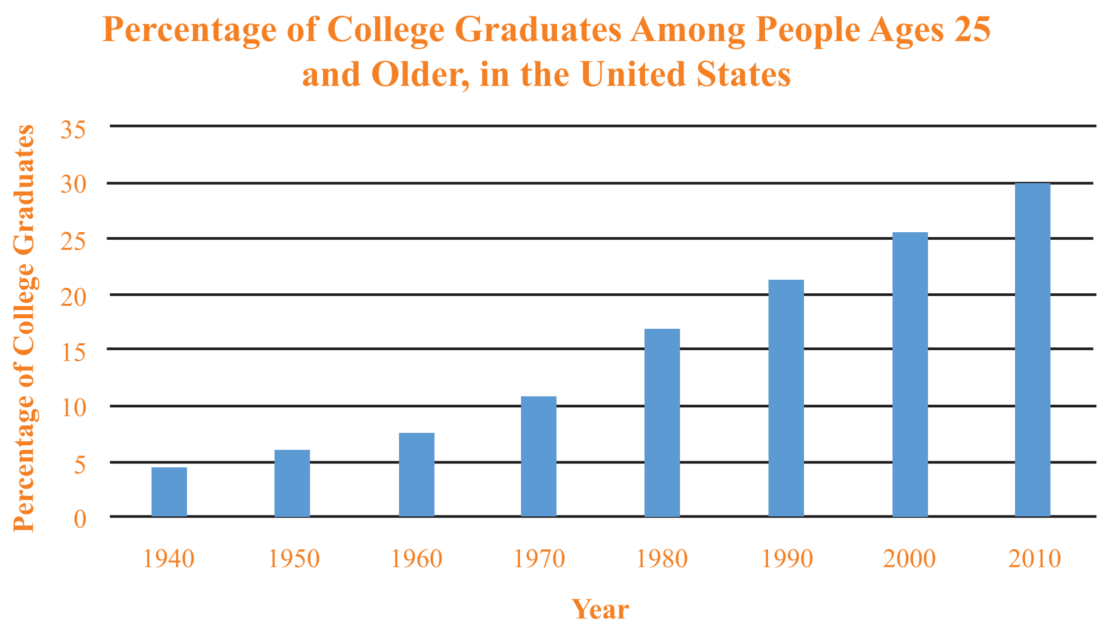
In section 1.2, you explored how to create a linear model for graphed data and use it to make estimates. In this activity, you will create your own bar graph, develop a model that estimates the relationship between the variables, and use your model to make a prediction.

Percentage of College Graduates Among People Ages 25 and Older, in the United States

Year	1940	1950	1960	1970	1980	1990	2000	2010
Percentage	4.6	6.2	7.7	11.0	17.0	21.3	25.6	29.9

Source: U.S. Census Bureau

Make a bar graph for the data in the chart above. Place the years on the horizontal axis and percentage of college graduates on the vertical axis. Be sure to give the graph a title and label the axes. Use a straightedge and graph paper to create neat lines. Tip: Refer to example 8 in section 1.2 for a similar problem.



1. Estimate the increase in the percentage of college graduates per year. Round your answer to the nearest tenth of a percent. Show all the steps you took to get your answer.

   Answers will vary, depending on data points chosen. One example: $\frac{29.9 - 4.6}{2010 - 1940} = \frac{25.3}{70} = 0.36$, which rounds to 0.4. This indicates that the percentage of college graduates increases by about .4 per year.

2. Now write a mathematical model (an equation) that estimates y, the percentage of Americans age 25 and older who graduated college x years after 1940.

   Answers will vary based on the calculation performed in #1. Continuing our example above: $y = 0.4x + 4.6$.

3. Using your model from step 2, project the percentage of college graduates for the year 2020. Again, show your steps.

   Answers will vary based on the equation obtained for #2. Continuing our example: $y = 0.4(80) + 4.6 = 36.6$. The projected percentage of college graduates in the year 2020 is 36.6.

Activity B: Figurate Numbers and Pascal's Triangle

Pythagoras of Samos, the man for whom the famed Pythagorean Theorem was named, was a mathematician in ancient Greece. He began a society called the School of Pythagoras, where students came to learn about math and Pythagoras's mystical beliefs. Among the teachings of the Pythagoreans was the idea that numbers were sacred and some numbers were more special than others. In this activity, we will explore certain "special" numbers called *figurate numbers* and look for patterns. (We will learn more about the life of Pythagoras in lesson 10.)

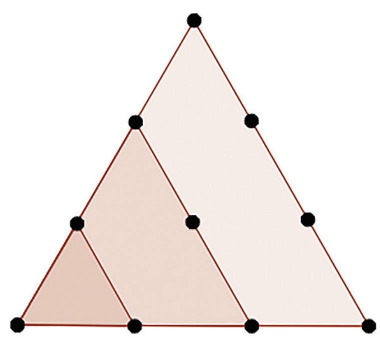
Figurate numbers are special because they can be represented by an arrangement of equally spaced dots in a regular geometrical shape. In other words, they are the number of evenly spaced dots needed to form both the outline and interior of a particular shape. You were introduced to figurate numbers in the Application Exercises for section 1.1. Now we will dig a bit deeper into the patterns formed by these special numbers. This activity has two parts.

Part 1

Important note: The number 1 is defined to be the first figurate number for all shapes, as a single point could form the basis for any shape.

1. If the pattern of the number of dots forms a triangle, then the number of dots is called a **triangular number**. The number 3 is a triangular number because 3 evenly spaced dots form a triangle. Likewise, the numbers 6 and 10 are also triangular numbers.

   Viewing tip: In the figure below, start by looking at the darkest triangle, which contains three dots. Now look at the triangle formed by overlapping the darkest triangle and the slightly lighter region. This second triangle contains six dots. Finally, look at the largest triangle, which contains all the points from the first two triangles. Notice that it contains ten dots, nine on its exterior and one on its interior.

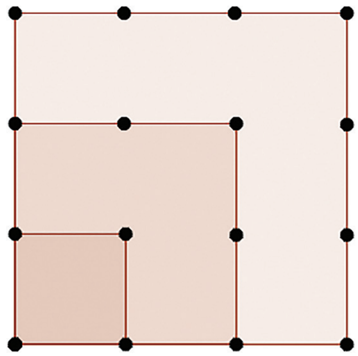
   Triangular numbers: 1, 3, 6, 10, . . .

   Look for a pattern in the first four triangular numbers listed in the caption above. (Hint: Note a pattern with the difference between each number and the following number.) Describe your pattern and find the next four triangular numbers. You may find it helpful to extend the figure so that larger triangles are formed. This will allow you to count the dots and verify the pattern.

   The pattern shows each triangular number is formed by adding the number of dots in its row to the number of dots in each previous row: 1 = 1, 3 = 1 + 2, 6 = 1 + 2 + 3, 10 = 1 + 2 + 3 + 4, 15 = 1 + 2 + 3 + 4 + 5, and so on. The next four triangular numbers are 15, 21, 28, and 36.

2. A pattern of dots that forms a solid square gives us the square numbers.

   Viewing tip: Start by looking at the darkest square in the figure below and notice that it contains four dots. Then look at the medium-sized square that overlaps the smallest square. This contains nine dots. Finally, look at the largest square formed by overlapping the smaller squares, and verify that it contains 16 dots, 12 on its exterior and 4 on its interior.

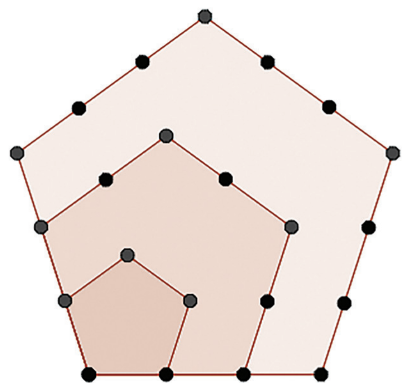


Square numbers: 1, 4, 9, 16, . . .

Look for a pattern in the first four square numbers listed in the caption above. Find the next four square numbers and describe your pattern.

In this pattern, each square number is formed by squaring the number of dots in its row: $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, and so on. The next four square numbers are 25, 36, 49, and 64.

3. Similarly, the pentagonal numbers are generated by the pattern of dots forming regular pentagons.

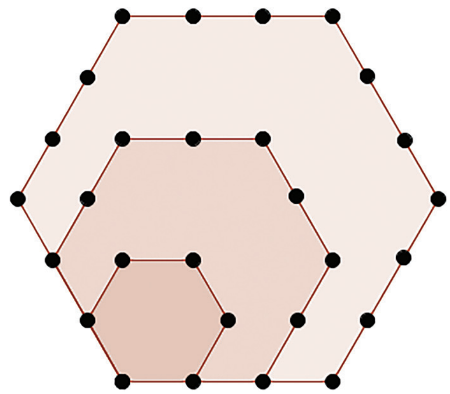


Pentagonal Numbers: 1, 5, 12, 22, . . .

Look for a pattern in the first four pentagonal numbers listed in the caption above. Find the next four pentagonal numbers and describe your pattern.

In looking for this pattern, note that the difference between the first two pentagonal numbers is 4, then the difference between the second and third pentagonal numbers is 7, then the next difference is 10. These differences increase by 3 each time. Following this pattern, we expect the next difference to be 13, and the following difference to be 16, and so on. The next four pentagonal numbers are 35, 51, 70, and 92.

4. The pattern of dots forming regular hexagons give us the hexagonal numbers.



Hexagonal Numbers: 1, 6, 15, 28, . . .

Look for a pattern in the first four hexagonal numbers listed in the caption above. Find the next four hexagonal numbers and describe your pattern.

This pattern is similar to the pentagonal number pattern, but note the difference between each listed hexagonal number and the next one. The differences are 5, 9, and 13. There is an increase of 4 for each difference. Following this pattern, we expect the next difference to be 17, then 21, and so on. The next four hexagonal numbers are 45, 66, 91, and 120. (Another pattern is that every other triangular number is a hexagonal number.)

Part 2

Once you have done that, you are ready to explore more patterns with the figurate numbers using Pascal's Triangle. Pascal's Triangle is a triangular arrangement of numbers that was named for mathematician Blaise Pascal (1623–1662), who studied it extensively. (We'll learn more about Blaise Pascal in lesson 11.) The triangle itself was discovered long before Pascal was even born, though; a version from thirteenth-century China has been found!

The arithmetic triangle, as Pascal's Triangle is also known, is constructed by placing the number one at the top of a page with two ones in the row beneath it. Each successive row is formed with one more entry than the row above it, a one at the very leftmost entry and the very rightmost entry. Each of the remaining entries is the sum of the two numbers directly above it. (See the diagram below.)

Pascal's Triangle has many interesting properties, enough to take up an entire course! For now, what we are interested in is the patterns formed by the figurate numbers in Pascal's Triangle. Your mission is to locate some of them.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

The first 11 rows of Pascal's Triangle

1. The diagram above shows the first 11 rows of Pascal's Triangle. Following the pattern, write down the 12th row.

   12th row of Pascal's Triangle: 1 11 55 165 330 462 462 330 165 55 11 1

2. The triangular numbers are the easiest to find in Pascal's Triangle. Using your list of triangular numbers from Part 1, look for them in the triangle. Use a red pen or pencil to circle the triangular numbers on the diagram. (If you need a hint, look at the diagonals.) Describe the pattern in Pascal's Triangle for triangular numbers. Does your pattern verify the next four triangular numbers you identified in Part 1, step 1?

   The 2nd diagonals should have all entries circled in red.

3. The hexagonal numbers are in a similar location in Pascal's Triangle. Using your list of hexagonal numbers from Part 1, look for them in the triangle. Use a blue pen or pencil to circle the hexagonal numbers on the diagram, making sure to also leave previous markings visible. Describe the pattern in Pascal's Triangle for hexagonal numbers. Does your pattern verify the next four hexagonal numbers you identified in Part 1, step 4?

   Every other entry in the 2nd diagonals should be circled in blue.

4. The square numbers and pentagonal numbers can also be found on Pascal's Triangle, but they are not quite as obvious because they require adding entries together in the triangle. To see how these work, and to learn about other applications of Pascal's Triangle to patterns of numbers, visit www.mathsisfun.com/pascals-triangle.html. (This link, along with all the other links mentioned in this course, can be found in clickable form on the Math Connections resource page on the Oak Meadow website at www.oakmeadow.com/curriculum-links. Bookmark that page for easy access to all the recommended online sources!)

Activity C: Logic Puzzles

Choose and complete one of the following activities. **Enrolled students:** Earn extra credit by doing both of these activities.

1. **Sudoku:** Read the explanation for solving a Sudoku puzzle at the top left of page 40 in your textbook. Copy the puzzle grid into your notebook and solve the puzzle by filling in the grid as described. Write a one-paragraph reflection on your experience with solving a Sudoku puzzle.

   Note: If you are not familiar with Sudoku puzzles and you want to start off by trying some easier puzzles, go to sudoku.com and choose "Beginner" from the "Difficulty" menu. A beginner-level puzzle will be generated for you to practice with. Scroll down to the bottom of the page for some great explanations and tips.

2. **Magic Squares:** Read the explanation and example for solving a Magic Square in the middle of the left-hand column on page 40 of the textbook. Complete exercise #48, parts a and b, on the same page. Write a one-paragraph reflection on your experience with solving a Magic Square.

   If you enjoy this type of puzzle, check out the links for Magic Square puzzles in the Additional Resources section at the end of the chapter. You can even learn how to create your own Magic Squares!

Going Further: Additional Resources

If you enjoy Sudoku puzzles, you may also enjoy Kenken and Kakuro puzzles. Check out the links for Sudoku, Kenken, and Kakuro puzzles in the Math Connections resource page on the Oak Meadow website www.oakmeadow.com/curriculum-links. You'll find other links and resources for this lesson there as well.

SHARE YOUR WORK

Once you have finished lesson 1, please submit the following items to your teacher:

- Journal entries A and B
- Activities A, B, and C
- Lesson 1 Test (from the test packet)

If you completed the Chapter 1 Test in the textbook for practice, do not include that with your submissions. Your teacher only needs to see the test from the test packet. You do not have to submit Practice Exercises or Application Exercises either.

Be sure your submission meets all criteria listed in your teacher's welcome letter. Refer to the course checklist for specific guidelines.

People in Mathematics

(Image credit: Math.info)

George Pólya (1887–1985) was a Hungarian mathematician who did not love mathematics in his youth but went on to literally write the book on problem-solving. *How to Solve It* has sold more than a million copies since it was published in 1945 and is still widely used today.

Archimedes of Syracuse (287–212 BCE) is considered one of the greatest mathematicians of all time. Many of his discoveries in geometry, such as the volume and surface area of a sphere, are still used today. He is most famous for solving problems through his inventions, and for a story in which he made a brilliant scientific discovery while in the bath and ran through the streets naked, yelling “Eureka!” To learn more about the legendary Archimedes, check out the three fascinating video links in the Math Connections resources page on the Oak Meadow website.

Lesson

2

Set Theory

When you were very young, you learned how to categorize objects into "sets," or collections, by characteristics such as size, shape, and color. As you have grown, so has your ability to classify abstract concepts using more sophisticated methods. In this lesson, we will explore ways of categorizing and representing sets, including the use of set notation and Venn diagrams. This will help you organize and make sense of information and enable you to more effectively solve problems.

This lesson should take approximately three weeks to complete.

Learning Objectives

- Represent a set using a description, the roster method, and set-builder notation.
- Distinguish between finite and infinite sets.
- Recognize subsets and the empty set, and use appropriate notation.
- Represent set relationships using Venn diagrams.
- Find the complement of a set, and the intersection and union of two sets.
- Perform operations with sets.
- Use Venn diagrams to solve problems.
- Apply knowledge of set operations and Venn diagrams to create and solve problems.
- Identify a mathematician of personal interest.

ASSIGNMENT CHECKLIST

- ☐ Mental Math Set A: Grouping Strategies
- ☐ Mental Math Set B: Multiplication Strategies
- ☐ Mental Math Set C: Division Strategies
- ☐ Read Chapter 2 in textbook.
- ☐ Complete a selection of exercises for sections 2.1 through 2.5.
- ☐ Read Chapter 2 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: How Big Is Infinity?
- ☐ Math Journal B: Activity Reflection
- ☐ Complete mathematician project topic proposal.
- ☐ Activity A: Blood Types and Venn Diagrams
- ☐ Activity B: Create Your Own Survey Problem

Why It Matters

> *"A set is a Many that allows itself to be thought of as a One."*
>
> **Georg Cantor**

Set theory involves classifying collections of information in order to understand relationships among the collections. This provides order and structure for many areas of mathematics, including algebra and topology, and is also useful for applications in computer science, biology, the social sciences, and business. For example, in algebra, when solving equations, we need to know which set of numbers we are working with—the real numbers, the integers, whole numbers, and so on—before we can find the solution. In computer science, programmers use set theory to set up searches of categorized information in databases. Whenever you enter a search term on the internet, set theory is at work behind the scenes!

Mental Math Warm-Ups

This lesson contains three sets of mental math warm-ups. Complete one set each week.

- Mental Math Set A: Grouping Strategies
- Mental Math Set B: Multiplication Strategies
- Mental Math Set C: Division Strategies

Mental Math Set A: Grouping Strategies

Can you calculate 274 + 198? Sure, you could pull out paper and pencil and add the three-digit numbers, carrying ones along the way. But there is an easier way! We can use the estimation techniques from lesson 1 to make exact calculation simpler.

If the problem had asked you to mentally calculate 274 + 200, it would not have been quite so daunting, would it? We would just add 2 to the hundreds column of 274, getting the answer 474. Notice, though, that 198 is awfully close to 200. In fact, 200 is a great estimate for 198. If we change the problem and add 200 instead of 198, we'll get an estimated answer that is pretty close to the true answer. But how close? Well, 200 is greater than 198 by 2 and we added it to 274 to get 474. That means our estimate is too high by 2. Therefore, if we subtract 2 from the estimated result of 474, we will get the exact solution 472.

Written mathematically, here is what we did:

$$274 + 198 = 274 + (200 - 2) = (274 + 200) - 2 = 474 - 2 = 472$$

We realized that 198 is equal to 200 – 2, so we substituted 200 – 2 for 198. Then we regrouped the numbers so we could easily add them.

In this week's mental math exercises, we will use similar grouping "tricks" to make it easier to solve problems mentally.

Day 1: With a partner, take turns giving each other an addition problem involving two-digit numbers. Make sure one of the numbers is close to an easy-to-work-with number. For example, 19 is a good choice because it is close to 20, so you might give the problem 25 + 19. Use the grouping trick to mentally find each sum and briefly explain to your partner how you changed the grouping of the original problem. For example, since 19 is 1 less than 20, we can mentally add 25 + 20, getting 45, and then subtract 1 since our approximation is too large by 1. Mathematically, it looks like this:

$$25 + 19 = 25 + (20 - 1) = (25 + 20) - 1 = 45 - 1 = 44$$

Gradually make the problems harder by using numbers with three, and perhaps even four, digits. If you do not have a partner, jot down several problems and see how many you can answer in two minutes.

Day 2: Today we will try the same activity with subtraction, but first let's look at whether it works the same way. Let's take, for example, 250 – 98. A good approximation for 98 is 100, so we can subtract 100 from 250 to get 150. We subtracted 100 instead of 98, so our answer is too large by 2. That means our answer is 152. Mathematically, we computed the following:

$$250 - (100 - 2) = (250 - 100) + 2$$

Notice that while we started with 100 – 2, when we regrouped we had to add 2 due to the distributive property over subtraction. Likewise, if we compute 250 – 102, we would subtract 2 because of the distributive property.

$$250 - (100 + 2) = 250 - 100 - 2 = 148$$

Repeat the exercise from Day 1 with subtraction problems. If it helps, use a calculator to quickly verify your answers.

Day 3: Does grouping also work for multiplication? Let's check it out. Suppose we want to find 200 × 21. Rather than doing two-digit multiplication, we can approximate the answer using 20 instead of 21. 200 × 20 is twice the value of 200 × 10, so we get 2 × 2,000, or 4,000. We know this approximation is lower than the true solution because we multiplied 200 by 20 instead of by 21. If we think of multiplication as groups of items, we calculated the number of items in 20 piles of 200 when we actually needed to find out how many items were in 21 piles of 200. That means we need to add one pile of 200 to our answer, so we get 4,000 + 200 = 4,200. Mathematically, we regrouped and applied the distributive property over addition like this:

$$200 \times 21 = 200 \times (20 + 1) = (200 \times 20) + (200 \times 1) = 4{,}000 + 200 = 4{,}200$$

Repeat the previous days' exercises with multiplication problems, being sure to use "convenient" numbers that make mental calculation simple.

Days 4 and 5: Repeat the previous three exercises, mixing things up with addition, subtraction, and multiplication problems. Challenge yourself or your partner by choosing slightly less convenient numbers.

Mental Math Set B: Multiplication Strategies

Imagine you are planning a party with your friends and you need to buy a dozen drinks that cost $1.50 each. How can you quickly tell how much the drinks will cost? In this week's mental math exercises, you will learn some techniques to help you easily solve this problem and other multiplication problems.

Day 1: To solve the problem with the drinks, a helpful multiplication trick is to double and halve. In other words, when you are finding the product of two factors, you can double one of the factors and halve the other. This is valid because multiplying the expression both by 2 and $\frac{1}{2}$ means we are actually multiplying by $\frac{2}{2}$, which is 1, and not really changing the problem. Choosing which factor is doubled and which is halved can make a difference in terms of how "nicely" the calculation works out. For instance, in our example we want to calculate 12 × $1.50. We could double 12 and halve $1.50, getting the product 24 × $0.75. But is that any easier to calculate than the original problem? Not really. However, if we double $1.50 and halve 12, we get 6 × $3, which is a much simpler problem to solve! We can easily see that our product is $18. With a partner, take turns giving each other multiplication problems to solve. Estimate as needed.

Day 2: Try using a trick to multiply prices that end in 99 or 98 cents. If you are buying 4 notebooks that each cost $2.99, we can calculate the price of 4 notebooks costing $3 to be $12, and then subtract the 4 extra cents, getting the total cost of $11.96. Mathematically, we used the distributive property over subtraction like this:

$$4 \times \$2.99 = 4(\$3.00 – \$0.01) = 4 \times \$3.00 - 4 \times \$0.01 = \$12.00 - \$0.04 = \$11.96$$

Practice this technique by making up problems to solve with your partner. The next time you are out shopping, use this mental math trick to figure out how much you will spend.

Day 3: Use "compatible" numbers to simplify multiplication problems with more than two factors. For example, if we need to multiply 20 × 15 × 5, we can change the order of the factors using the associative property of multiplication to first multiply 20 and 5 to get 100, and then multiply 100 by 15, getting 1,500. Since 20 × 5 equals 100, an easy-to-work-with number, 20 and 5 are "compatible." Compatible numbers don't have to yield a product of 100; any easy-to-work-with product can be used for this trick. With a partner, practice giving each other problems with three factors, some of which are "compatible."

Day 4: If none of your factors is compatible, you may be able to break apart the factors into other factors in order to find compatible numbers. For example, 75 × 12 doesn't give us compatible factors, but if we rewrite 75 as the product of 3 and 25, and 12 as the product of 3 and 4, our problem then becomes 3 × 25 × 3 × 4, and by the associative property of multiplication, we can multiply the compatible numbers 25 and 4, giving us 100 × 3 × 3 = 900. With a partner, pose problems to each other with numbers that can be broken down to form compatible numbers.

Day 5: Practice multiplying numbers using all the techniques you learned in the previous four days.

Mental Math Set C: Division Strategies

> *"Science attempts to find logic and simplicity in nature. Mathematics attempts to establish order and simplicity in human thought."*
>
> **Edward Teller**

Division problems come up all the time in real life. When you grab a bite to eat with friends, how do you share the cost? When you order a pizza, how do you split up the slices so everyone gets an equal amount? Which size bottle of shampoo is a better value? Having some tricks on hand will help make mental division quick and easy.

Day 1: Practice division by 10, 100, 1,000, and other positive powers of 10. Division by a positive power of 10 is simple because we can just move the decimal point one position to the left for each 0 in the divisor. For example, a $45.00 restaurant bill divided among 10 people would cost $4.50 per person. With a partner, take turns giving each other word problems that require division by a positive power of 10.

Day 2: What if the numbers in a real-life division problem aren't quite as "nice" as the powers of 10? Our estimating tricks from lesson 1 can help! Approximate both the dividend and divisor, and then solve. It is helpful to round up for either the dividend or divisor and then round down for the other in order to reduce the total error from estimating the answer. For example, if we are finding the unit price of a 96 fluid ounce bottle that costs $16.29, we can round 96 up to 100 and round $16.29 down to $16, giving us an estimated quotient of 16 cents per ounce. With a partner, take turns posing problems and using estimated quantities in order to mentally do division.

Day 3: Quick, what's 410 divided by 5? Now that you know about the trick for dividing by 10, you can also easily divide by 5! If we were to divide 410 by 10, we would quickly come up with 41. Since 10 is twice as much as 5, our result of 41 is half of what we would get by dividing 410 by 5. In other words, we divided by twice as much as we should have. To correct for this, we can double our result of 41 to get the answer 82. This "divide by 10, then double" rule will always work for division by 5. Mathematically, we substituted $\frac{10}{2}$ for 5:

$$\frac{410}{5} = \frac{410}{\frac{10}{2}} = \frac{410}{\frac{10}{1} \times \frac{1}{2}} = \frac{410}{10} \times \frac{2}{1} = 41 \times 2 = 82$$

With a partner, take turns giving each other problems with division by 5 and solving them by dividing by 10, then doubling. Bonus question: would it also work to double first, then divide by 10? Test it out and see for yourself!

Day 4: Division by 20 can be done by first dividing by 10, and then dividing the result by 2. For example, in order to divide 640 by 20, we can calculate 640 ÷ 10 = 64, and then 64 ÷ 2 = 32. Using math notation, the process looks like this:

$$\frac{640}{20} = \frac{640 \times 1}{10 \times 2} = \frac{640}{10} \times \frac{1}{2} = 64 \times \frac{1}{2} = 32$$

With a partner, take turns giving each other problems with division by 20 and solving them by dividing by 10, then 2.

Day 5: Expand on the ideas above to practice dividing by 50, 200, 500, etc. How can you easily divide by 50? (Hint: How could the trick for division by 5 be adjusted to work for 50?) Use the estimation techniques from Day 2 if the quotient is not a whole number.

> *"Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country."*
>
> **David Hilbert**

Assignments

Textbook Assignments and Test

1. Read textbook sections 2.1 through 2.5. For each section, follow along with the examples and try the Checkpoint problems. Check your answers with the back of the book. Verbally answer the Concept and Vocabulary Check exercises at the end of the section. Check your answers with the back of the book.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 2.1 through 2.5 (odd-numbered problems only). Choose several problems of each type to ensure sufficient practice.
3. Do all odd-numbered Application Exercises for each Exercise Set. Check your answers with the back of the book. Make any necessary corrections and review areas that need work. If you need additional practice, you may want to complete a selection of even-numbered problems.
4. Review the Chapter 2 Summary at the end of the chapter. If you feel you need additional practice, select problems from the Chapter 2 Review.
5. Complete the Chapter 2 Test from the textbook (for independent students) or the Lesson 2 Test from the test packet (for enrolled students). Students who complete the textbook test are encouraged to check their answers in the back of the book, making necessary corrections and reviewing areas that need work. Students who are enrolled in Oak Meadow School must complete the test from the test packet.

Math Journal

Complete both journal assignments (one per week).

- Journal A: How Big Is Infinity?
- Journal B: Activity Reflection

Journal A: How Big Is Infinity?

In Section 2.1 you learned that a finite set is a set with cardinality 0 or a natural number, which means that the set contains 0 or a natural number of elements. A set that is not finite—in other words, its cardinality is not 0 or a natural number—is called an infinite set. There are an infinite number of infinite sets, but are all infinite sets equivalent (that is, the same size)?

Find out more about infinite sets by watching one (or both) of the following videos.

> Dennis Wildfogel's "How Big Is Infinity?" (TED-Ed):
> ed.ted.com/lessons/how-big-is-infinity#watch
>
> Numberphile's "Infinity Is Bigger than You Think":
> www.youtube.com/watch?v=elvOZmod4Ho

In your math journal, write a one- or two-paragraph response describing what you learned about infinite sets and sharing your thoughts on the video(s). Be sure to mention which video you watched.

Journal B: Activity Reflection

Do this journal assignment after you complete Activity B (see the activity section of this lesson).

Think back for a minute about how you created your survey problem for Activity B. Write a one paragraph reflection on your experience. Some questions to consider: Was this activity easier or more challenging than you initially thought? Did you encounter any difficulties as you created your survey problem? If so, what did you do to adjust?

Project Milestone

Mathematician Project Topic Proposal

After completing lesson 6, you will wrap up the first semester by writing a paper for your midterm project. This week, you will begin working on your project by creating a proposal.

For your midterm project, you will write a three- to five-page research-based paper on a mathematician of your choice. But there's a twist: the paper must be written from the point of view of your chosen mathematician, as if they were writing an autobiography. This will allow you to "get into the head" of your mathematician, like a character actor, and relive their life and mathematical discoveries. While the paper must be grounded in fact and will require citations, you have full creative license to interpret the circumstances surrounding the facts and imagine your mathematician's thoughts and feelings. You may choose to write the paper in the form of a letter, diary entries, a memoir, an interview, or even an obituary "written" by your mathematician. Be creative!

This paper should focus on the mathematical work done by your chosen person. Information such as their early life and education is important, but the spotlight should be on the person's mathematical ideas. You are not expected to fully understand this person's work, but you should be able to explain it in general terms.

Since this is a large project, it is best approached by researching, organizing, and writing the paper in stages over the course of the semester, culminating with the final submission after lesson 6. There will be three "milestone assignments" along the way: the topic proposal, bibliography, and outline.

Note for enrolled students: Each milestone assignment must be approved by your teacher before you move on to the next phase of the project.

In this lesson, you will accomplish the first milestone: choosing a mathematician to write about. The list below contains some popular suggestions, but you are free to select any mathematician. For inspiration, flip through this coursebook and look at the "People in Mathematics" spotlights found in each lesson. There is also an extensive index of mathematicians on the MacTutor History of Mathematics Archive at mathshistory.st-andrews.ac.uk/Biographies/.

You are encouraged to choose a mathematician who is new to you, but please make sure you can find sufficient information to write a three- to five-page paper. If you are up for a challenge, choose a mathematician who is female, or from another culture, or someone who is alive and working today.

For this lesson's milestone, select your mathematician and do some preliminary research. This will let you get to know your mathematician and assess whether enough information is available for you to write a paper about that person. Write a two-paragraph topic proposal that includes:

- The mathematician's name and a sentence or two describing this person's mathematical contributions in your own words.
- An explanation of why you selected this person.
- A citation of the source(s) of your preliminary research information.

Important reminder for all students: Use only reputable sources written by an authority on the subject. The MacTutor History of Mathematics Archive mentioned above is an excellent starting point. Reputable encyclopedias and dictionaries of scientific biography are fine too. Please note that sites like Wikipedia are never acceptable sources for research information. In most cases, student and teacher webpages are not acceptable either. Also avoid blog posts (unless written by an authority on the subject) and popular TV station sites like bio.com. If you have a question about whether a source is reputable, ask your parent, tutor, or teacher.

Some suggested mathematicians (remember, you are not limited to people on this list):

Muhammad al-Kwarizmi

Archimedes

George Boole

Georg Cantor

Charles Dodgson (aka Lewis Carroll)

René Descartes

Albert Einstein

Paul Erdös

Leonhard Euler

Pierre de Fermat

Leonardo Fibonacci

Leonardo da Vinci

Galileo Galilei

Carl Friedrich Gauss

Sophie Germain

Stephen Hawking

Omar Khayyám

Ada Lovelace

John Forbes Nash Jr.

Isaac Newton

Emmy Noether

Blaise Pascal

Plato

Pythagoras

George Pólya

Srinivasa Ramanujan

Alan Turing

Activities

Complete the activities below.

- Activity A: Blood Types and Venn Diagrams
- Activity B: Create Your Own Survey Problem

Activity A: Blood Types and Venn Diagrams

The discovery and classification of different antigens in blood revolutionized the field of medicine. Reread the information about blood types in the Blitzer Bonus box on page 91 of *Math for Your World* and refer to the Venn diagram in Figure 2.23. Note, in particular, that the Venn diagram classifies the eight different blood types by the presence or absence of each of the three antigens A, B, and Rh in red blood cells. Also note that in order to receive blood in a transfusion, "the recipient must have all or more of the antigens present in the donor's blood." This means that "the set of antigens in a donor's blood must be a subset of the set of antigens in a recipient's blood." (Blitzer 95)

Let's consider, for example, that Marina, who has type A+ blood, needs a blood transfusion. To have type A+ blood, Marina must have antigens A and Rh, but not B, present in her blood. Because she needs to have all or more of the antigens in the donor's blood, Marina cannot receive blood that contains B antigens. Looking at the Venn diagram, we see that this limits Marina to receiving blood only from donors with O+, O–, A+, or A– blood type.

Use the information provided on page 91 to fill in the chart below and then answer the questions that follow.

Recipient Blood Types	Compatible Donor Blood Types
A+	O+, O–, A+, A–
B+	**O+, O–, B+, B–**
AB+	**O+, O–, A+, A–, B+, B–, AB–, AB+**
O+	**O+, O–**
A–	**A–, O–**
B–	**B–, O–**
AB–	**AB–, A–, B–, O–**
O–	**O–**

1. A universal recipient is a person who can receive blood from a donor with any blood type. Based on the Venn diagram and your chart above, which blood type does a universal recipient have?

   AB+

2. A universal donor is a person who can donate blood to a person with any blood type. Based on the Venn diagram and your chart above, which blood type does a universal donor have?

   O–

3. Nikki, who has blood type B–, is donating blood. What blood type(s) must a recipient be to receive her blood?

   B+, B–, AB+, or AB–

4. Miguel was in a serious accident and needs a blood transfusion. A quick test at the ER lab indicates that he has O+ blood. What donor blood type(s) can he receive?

   O+ or O–

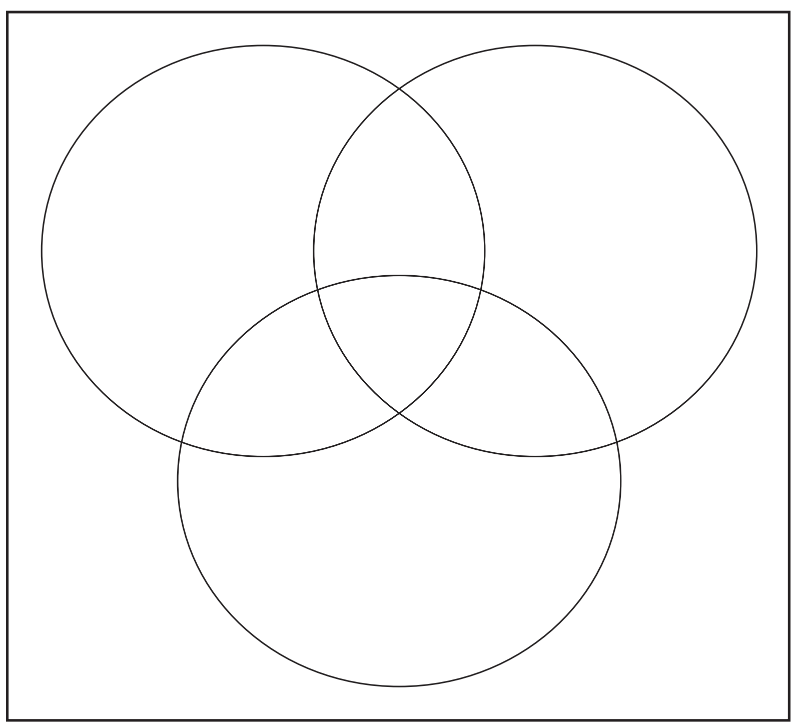
Activity B: Create Your Own Survey Problem

Now it's your turn! Make up your own three-set survey problem about a fictional situation of your choice. Note: This is fictional, so you will not actually conduct a survey.

For the sake of simplicity, assume that you will survey 100 fictional participants. Start by creating a three-set Venn diagram and fill in fictional numbers so that all 100 participants are represented. Be sure to consider including participants in the region outside of the three sets, and don't forget that the intersections of the sets are included in the total number for each set. Remember, the sum of the numbers in all of the individual regions on the Venn diagram should be 100.

The next step is to determine how much information you must present in order to make the problem solvable. What questions will you ask? For inspiration, refer to similar problems in the Application Exercises on pages 103–104 of the textbook, but be sure that you create a unique problem that will require the solver to make a three-set Venn diagram.

Include at least three questions pertaining to your problem. Also provide the solution to your problem, including a completed and labeled Venn diagram illustrating your problem. You may use or copy the empty Venn diagram below.



If possible, briefly explain to a friend or family member how to solve survey problems and have them test out your problem. This will help you determine if you provided enough information to create a Venn diagram and answer your questions.

Answers will vary, but in evaluating the student-created problem, make sure that it is solvable and that the Venn diagram supports the solution. The numbers in each section of the Venn diagram, including the part outside the rings, should sum to 100. A common mistake made by students is to not subtract the numbers in the intersections that should have already been counted if starting from the center of the diagram.

Going Further: Additional Resources

For additional resources on solving word problems using Venn diagrams, games involving set theory, and more, visit the resource page on the Oak Meadow website.

SHARE YOUR WORK

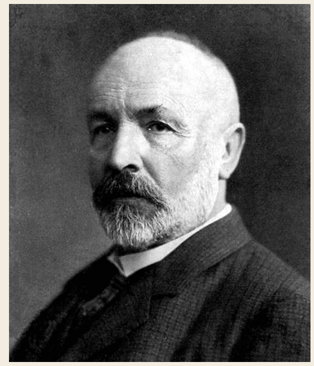
After you submit your topic proposal for your midterm project, please wait for teacher approval before moving on with your research. Once you hear back from your teacher, you should immediately begin searching for sources. This will allow plenty of time for any books you order or request via interlibrary loan to arrive in time to submit your bibliography in lesson 4 (the next project milestone).

Once you have finished lesson 2, please submit the following items to your teacher:

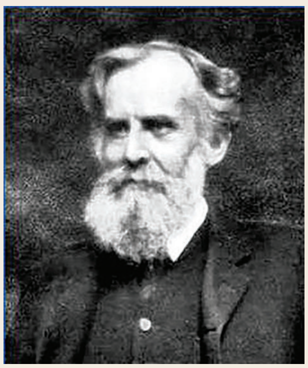
- Journal entries A and B
- Mathematician Project Topic Proposal
- Activities A and B
- Lesson 2 Test (from the test packet)

Remember, you do not need to submit any Practice or Application Exercises. Be sure your submission meets all criteria listed in your teacher's welcome letter. Refer to the course checklist for specific guidelines. Please feel free to contact you teacher if you have any questions.

People in Mathematics



Georg Cantor (1845–1918) was a Russian mathematician who is considered the father of set theory. He proved that some infinities are bigger than other infinities. To see this mind-blowing proof in action, check out Vi Hart's video link in the online resources for this lesson.



John Venn (1834–1923) was a British mathematician and logician. He came up with a way to visually represent sets using intersecting discs. Today we refer to his creation as a "Venn diagram."

Emmy Noether (1882–1935) was determined to study mathematics at a time when women in Germany were not permitted to officially do so. She studied anyway and eventually earned her doctorate in mathematics and obtained positions teaching mathematics at several universities. Her work in abstract algebra and theoretical physics earned the respect of other prestigious academics, including Albert Einstein. When removed from her job by Nazis because she was Jewish, she again persevered and continued her teaching and research in the United States.

Lesson 6

Polynomials, Quadratic Equations, and Quadratic Functions

In this lesson, we will begin with factoring polynomials and then dig into quadratic equations and functions. We will cover three methods of solving quadratic equations and then look at several real-life applications of quadratic equations. While some of the material, like factoring, will likely be review, other sections of this lesson may touch on concepts that are new to you. If you find that you need additional practice and examples, be sure to check the supplemental resources for this lesson on the Oak Meadow website.

This lesson should take approximately three weeks to complete.

Learning Objectives

- Add, subtract, and multiply polynomials.
- Model situations with polynomial functions.
- Factor out the greatest common factor of a polynomial and factor by grouping.
- Factor trinomials and the difference of squares.
- Solve quadratic equations by factoring, using the square root property, and the quadratic formula.
- Determine the most efficient method for solving quadratic equations.
- Solve problems using quadratic equations.
- Recognize characteristics of parabolas.
- Find the vertex and intercepts of a parabola.
- Graph quadratic functions.
- Solve problems involving quadratic functions.

ASSIGNMENT CHECKLIST

- ☐ Mental Math Set A: Pair of Numbers Game
- ☐ Mental Math Set B: Difference of Two Squares Multiplication Trick
- ☐ Mental Math Set C: Visualizing Graphs of Quadratic Equations
- ☐ Read Chapter 6 in textbook.
- ☐ Complete a selection of exercises for sections 6.1 through 6.5.
- ☐ Read Chapter 6 Summary.
- ☐ Complete test from textbook OR test packet.
- ☐ Math Journal A: Create a Word Problem
- ☐ Math Journal B: Choosing a Strategy for Solving Quadratic Equations
- ☐ Activity A: Fibonacci Numbers, Golden Rectangles, the Golden Ratio, and the Golden Spiral
- ☐ Activity B (optional): De Morgan's Age Problem

- Construct a Golden Rectangle.
- Use quadratic solving techniques to calculate the actual value of the Golden Ratio.

Why It Matters

Quadratic equations help model various real-world problems such as where a thrown baseball will land, what blood pressure a person of a certain age should have, and which dimensions you should choose for your dog run so your dog has the largest possible space to play in for the amount of fencing you have. Many other applications of quadratic equations exist in science, engineering, and business. Learning how to factor and use the quadratic formula provides us with helpful methods of solving and understanding quadratic equations, as does knowing how to graph them.

> *"If I have seen further than others, it is by standing upon the shoulders of giants."*
>
> **Isaac Newton**

Mental Math Warm-Ups

This lesson contains three sets of mental math warm-ups. Complete one set each week.

- Mental Math Set A: Pairs of Numbers Game
- Mental Math Set B: Difference of Two Squares Multiplication Trick
- Mental Math Set C: Visualizing Graphs of Quadratic Equations

Mental Math Set A: Pair of Numbers Game

In this week's mental math exercise, you will play a game with your partner that will help you to mentally factor polynomials. In this game, one player will choose a pair of integers and tell the other player the sum and product of those numbers. The goal is for the other player to mentally figure out the original numbers.

For example, if you choose the numbers 3 and –4, you would tell your partner that the sum of your numbers is –1 and the product is –12. Your partner would then try to figure out what factors of –12 could sum to –1, and hopefully come up with 3 and –4.

Day 1: With a partner, take turns thinking of a pair of numbers and giving the other player the sum and product of those numbers. To start out, choose integers between –5 and 5.

Days 2 and on: Repeat the game but make it more challenging by choosing numbers outside of the range –5 to 5. Gradually increase the difficulty of your chosen numbers. You might select numbers whose product is an integer with many factors, such as 36 or 64.

Mental Math Set B: Difference of Two Squares Multiplication Trick

In this lesson, we learned how to factor the difference of two squares. We can apply that principle to come up with a neat mental multiplication trick!

Let's start by choosing two numbers that are an equal distance away from a "convenient" number (like a multiple of 10). For example, let's choose the numbers 15 and 25, which are both 5 units away from the number 20. To find their product, we could use some of our previous mental math tricks, but let's try something different. Let's think of 15 as 20 – 5 and 25 as 20 + 5. Can you see where this is going? When we take the product of 20 – 5 and 20 + 5, we have (20 – 5)(20 + 5), which looks an awful lot like our formula for the difference of two squares:

$$a^2 - b^2 = (a + b)(a - b)$$

From this formula, we know the following:

$$(20 - 5)(20 + 5) = 20^2 - 5^2$$

With our strengthened mental math abilities, we can easily solve that!

$$20^2 - 5^2 = 400 - 25 = 375$$

In summary, when we multiply two numbers that are an equal distance from a convenient number, we can simply subtract the square of that equal distance from the square of the convenient number. Presto!

Day 1: Try the difference of two squares trick a few times, using a calculator afterward to verify your answers. To start, find 98 × 102. Create your own problems. Look for situations where the problem is very easy to solve and notice when the problems are more challenging to do mentally. What makes them easy or challenging?

Day 2: Teach this trick to a friend or parent, and also explain why it works. Walk them through a few examples.

Days 3 and on: Create more challenging examples and see how far you can stretch your mental math muscles.

Mental Math Set C: Visualizing Graphs of Quadratic Equations

In lesson 5, we visualized graphs of linear equations. This week, we will extend those exercises to include visualizing graphs of quadratic equations.

Days 1 and on: Write two or three quadratic equations in standard form ($ax^2 + bx + c = 0$) and with a partner, take turns examining each of the equations. What is the value of a? If a is positive, then the graph of the equation, a parabola, will open in an upward direction, like a smile. If a is negative, then the parabola will open in a downward direction, like a frown. Now look at c, which is the y-intercept. This number (including the sign) indicates where the parabola will cross the y-axis. Write down the information you collected so far.

The axis of symmetry is the vertical line $x = \frac{-b^2}{a}$ where a and b are the values of the coefficients in the equation's standard form. The parabola forms a mirror image on either side of this vertical line. The vertex must fall on this line, so if the axis of symmetry is negative, the vertex will be to the left of the y-axis. Likewise, if the axis of symmetry is positive, the vertex will be to the right of the y-axis. Write down the axis of symmetry on your list of information.

Now, looking at the information you wrote down, visualize and describe out loud what this parabola looks like based on whether it faces upward or downward, the location of its axis of symmetry, and the location of its y-intercept. What quadrant do you think the vertex will be in? (This isn't always clear, but you can make a good guess from just the information you have.) Have your partner use a graphing calculator, if one is available, to check whether your description is accurate. Continue to practice this exercise each day this week. The more you practice it, the better you will get at visualizing and graphing parabolas.

Assignments

Textbook Assignments and Test

1. Read textbook sections 6.1 through 6.5. Follow along with the examples and try the Checkpoint problems, checking your answers with the back of the book. Verbally answer the Concept and Vocabulary Check exercises at the end of the section and check your answers.
2. After reading each textbook section, complete a selection of problems from each section of the Practice Exercises 6.1 through 6.5 (odd-numbered problems only). Choose several problems of each type.
3. Do all odd-numbered Application Exercises for each Exercise Set (6.1 through 6.5). Check your answers with the back of the book. Make any necessary corrections and review areas that need work.
4. Review the Chapter 6 Summary at the end of the chapter. Use the Chapter 6 Review for extra practice, if necessary.
5. Complete the Chapter 6 Test from the textbook (for independent students) or the Lesson 6 Test from the test packet (for enrolled students). Students who complete the textbook test are encouraged to check their answers in the back of the book, making necessary corrections and reviewing areas that need work. Students who are enrolled in Oak Meadow School must complete the test from the test packet.

> *"It is not knowledge, but the act of learning, not the possession of but the act of getting there, which grants the greatest enjoyment."*
>
> **Carl Friedrich Gauss**

Math Journal

Complete both journal assignments (one per week).

- Math Journal A: Create a Word Problem
- Math Journal B: Choosing a Strategy for Solving Quadratic Equations

Journal A: Create a Word Problem

In the examples and Application Exercises, you have seen some word problems that relate to quadratic equations. Using these problems as inspiration, create your own original word problem whose solution involves a quadratic equation. Keep the problem simple. Also briefly describe how you would go about solving your problem. You do not need to actually solve the problem, but rather explain what kinds of steps you would need to take in order to solve it.

Journal B: Choosing a Strategy for Solving Quadratic Equations

In Chapter 6, we learned three different strategies for solving quadratic equations: solving by factoring, solving by using the quadratic formula, and solving by the square root property. Explain how you decide which strategy would be most efficient for you to use. Give an example of a quadratic equation that would be a good choice for each of the three strategies, and state why you would choose that method.

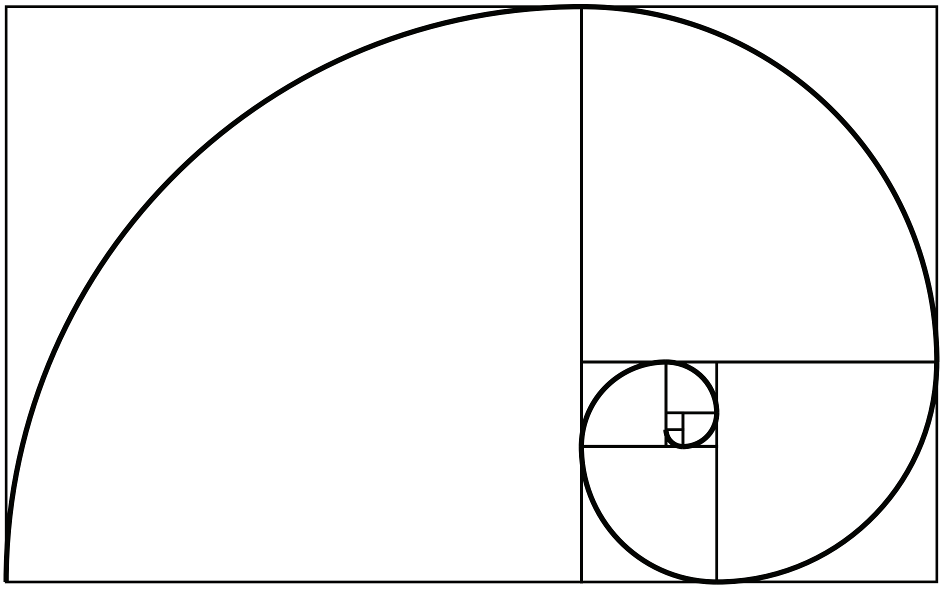
Examples will vary. The following are typical explanations:

- **When a quadratic equation has only a term for the variable squared, not the variable to the first power, the square root property is a good choice of method.**
- **When a quadratic equation is easily recognizable as being factorable using one of our factoring techniques, factoring is a good choice of method.**
- **When a quadratic equation does not fall into one of the above categories, the quadratic formula is a good choice of method.**

Activities

Complete Activity A below. Activity B is optional (enrolled students will receive extra credit for completing Activity B).

- Activity A: Fibonacci Numbers, Golden Rectangles, the Golden Ratio, and the Golden Spiral
- Activity B (optional): De Morgan's Age Problem



Activity A: Fibonacci Numbers, Golden Rectangles, the Golden Ratio, and the Golden Spiral

"Do not worry too much about your difficulties in mathematics, I can assure you that mine are still greater."

Albert Einstein

Back in lesson 3, you were introduced to the Fibonacci Sequence and the Golden Ratio through two videos. In this lesson, we will revisit these topics by creating our own Golden Rectangles and Golden Spiral, and calculating the Golden Ratio for ourselves. We will do this in two ways (complete both part 1 and part 2 of this activity).

Part 1: Make a Fibonacci Rectangle and approximate the Golden Ratio

First, watch Vi Hart's video, "Doodling in Math: Spirals, Fibonacci, and Being a Plant (1 of 3)." Pay particular attention to the part where she creates rectangles and a spiral based on the Fibonacci numbers (from 1:10 to 1:40).

www.khanacademy.org/math/recreational-math/vi-hart/spirals-fibonacci/v/doodling-in-math-spirals-fibonacci-and-being-a-plant-1-of-3

Now it's your turn! On a piece of graph paper, follow Vi's steps to create your own rectangles and spirals. (Please note that the smaller rectangles are rough approximations of a Golden Rectangle, while the larger ones get closer and closer to the actual proportions.)

Next, we will approximate the Golden Ratio using the Fibonacci numbers. Start by writing out the first 12 terms of the sequence. Remember, the Fibonacci sequence begins with two 1s and then each subsequent term is found by adding the two terms before it. It begins: 1,1,2,3,5,8, . . . List these first ten terms in the first column of the chart below, continuing where the examples left off.

Now write the ratio of each term to the previous term in the second column of the chart. Remember, a ratio is simply a fraction. Each ratio will have the Fibonacci number from that row divided by the Fibonacci number in the previous row.

Using a calculator, divide the numbers in the ratios in the second column and record the results in the appropriate row in the third column. Keep 10 decimal places for a high level of accuracy. These values represent the approximation for Phi. Notice that as we divide larger Fibonacci numbers, we are getting closer and closer to a particular irrational number, Phi, which is the Golden Ratio. The value of the ratios alternate between being slightly higher than Phi and slightly lower than Phi, but we can close in on a great approximation if we use large enough successive Fibonacci numbers.

Fibonacci number	Ratio of this term to the previous term	Resulting ratio (approximation for Phi)
1	–	–
1	1	= 1
2	$\frac{2}{1}$	= 2
3	$\frac{3}{2}$	= 1.5
5	$\frac{5}{3}$	= 1.6666666667
8	$\frac{8}{5}$	= 1.6
13	$\frac{13}{8}$	= 1.625
21	$\frac{21}{13}$	= 1.6153846
34	$\frac{34}{21}$	= 1.6190476
55	$\frac{55}{34}$	= 1.6176471
89	$\frac{89}{55}$	= 1.6181818
144	$\frac{144}{89}$	= 1.6179775

Part 2: Make a true Golden Rectangle and calculate the exact value of the Golden Ratio

Now that we have come up with an approximation for Phi, the Golden Ratio, and we have a good idea of what a Golden Rectangle should look like, we will construct a true Golden Rectangle and calculate the exact value of Phi.

First, read the Blitzer Bonus on the bottom of page 406 to learn more about the Golden Rectangle.

Next, watch the following video, "How to Make a Golden Rectangle and Golden Spiral," which demonstrates how to construct a Golden Rectangle and Golden Spiral using a straightedge and compass.

www.youtube.com/watch?v=TLxmLooZlg8

It's your turn again! Follow the same steps in the video demonstration to create your own Golden Rectangle and Golden Spiral on a sheet of paper using only a straightedge and compass. (If you need to refresh your memory on how to construct the original square and perpendicular lines, please refer to the resource page on the Oak Meadow website.)

Next, complete all parts of Application Exercise 87 on page 408 in your textbook in order to obtain an exact answer for Phi, the Golden Ratio. Show all of the steps in setting up your proportions and solving the resulting quadratic equation.

Finally, compare the exact value of Phi to the final estimate you obtained in the chart for Part 1. How close was the closest approximation? How many decimal places were recorded accurately in your approximation?

Application Exercise #87:

a. $\frac{1}{\Phi - 1}$

b. $\frac{\Phi}{1} = \frac{1}{\Phi - 1}$

$\Phi(\Phi - 1) = 1$

$\Phi^2 - \Phi = 1$

$\Phi^2 - \Phi - 1 = 0$

$\Phi = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2}$

We can't have a negative ratio here, so we only consider the solution $\Phi = \frac{1 + \sqrt{5}}{2}$

c. $\frac{1 + \sqrt{5}}{2}$

Comparing our approximation of Phi in Part 1 to the exact value of Phi from Part 2, we find that they are very close. $\frac{1 + \sqrt{5}}{2} \approx 1.6180339887$**. The last approximation we calculated was 1.6179775, which is accurate to the hundredths place. The previous approximation we calculated was 1.6181818, which is accurate to the thousandths place. If we were to calculate a few more successive ratios of Fibonacci terms, we would obtain an even better approximation for Phi.**

Activity B (optional): De Morgan's Age Problem

In the nineteenth century, famed mathematician Augustus De Morgan declared that he was x years old in the year x2. What age and year did Augustus De Morgan refer to, and in what year was he born? Hint: Trial and error within some logical framework is a good strategy here.

The problem mentions that it was posed in the 19th century, which gives us a clue about where to start. The year he was referring to could have occurred in the 1800s, or possibly even in the late 1700s (depending on when in the 19th century this occurred). If it happened in the 1800s, then we're looking for a year that is a perfect square between 1800 and 1899. The square root

of 1800 is 42.426 and the square root of 1899 is 43.578, so the square root we are looking for is between these two values. 43 is the only whole number that fits. That would make Augustus De Morgan 43 in the year $43^2 = 1849$ and means he would have been born in 1849 – 43 = 1806. This fits the problem's specifications!

Going Further: Additional Resources

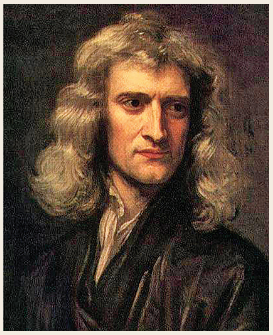
For additional resources about quadratic equations, graphing quadratic functions, Golden Rectangles and Spirals, and more, visit the resource page on the Oak Meadow website.

SHARE YOUR WORK

Once you have finished lesson 6, please submit the following items to your teacher:

- Journal entries A and B
- Activity A
- Optional: Activity B (to earn credit for your solution, you must include a clear write-up of your steps)
- Lesson 6 Test (from the test packet)

People in Mathematics



Isaac Newton (1643–1727), according to legend, discovered gravity when an apple from a tree dropped onto his head. (To learn more about whether this story has any truth to it, check out the online resources for this lesson.) Newton did, indeed, define the law of universal gravitation as well as the three laws of motion. He also made revolutionary contributions to the fields of optics, astronomy, and mathematics with his development of calculus.

Johann Carl Friedrich Gauss (1777–1855) started life as a child prodigy in what is now Germany. In elementary school, he was given an assignment by his teachers to keep him busy: to sum the numbers from 1 to 100. He accomplished this task very quickly by recognizing that there were 50 pairs of numbers that summed to 101. (For example, 1 + 100, 2 + 99, and so on.) Gauss made significant contributions to number theory, geometry, and geodesy (the study of the shape and size of the Earth).

Srinivasa Ramanujan (1887–1920) was a self-taught mathematical genius from India. As a young adult, he dealt with recurring illness and the shame of not succeeding in college due to his neglect of all subjects besides mathematics. Ramanujan reached out to mathematician G. H. Hardy, who was so impressed by Ramanujan's mathematical work that he arranged for the young man to join him in England. Both independently and together with Hardy, Ramanujan made significant advances in the fields of number theory and mathematical analysis until his death at the age of 32.

Appendix